Math for America minicourse

Introduction to Category Theory

Session 1: What is abstract math?

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Plan

Session 1: What is abstract math?

Session 2: Sameness

Session 3: Universal properties

Plan for today

1. Introduction
2. Abstraction and analogies
3. Relations
4. Categories
5. Examples: numbers, symmetries, quadrilaterals
6. Examples: factors and privilege.

1. Introduction

Pure mathematics is a framework for agreeing on things.

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pure mathematics

↓

applied mathematics

↓

science

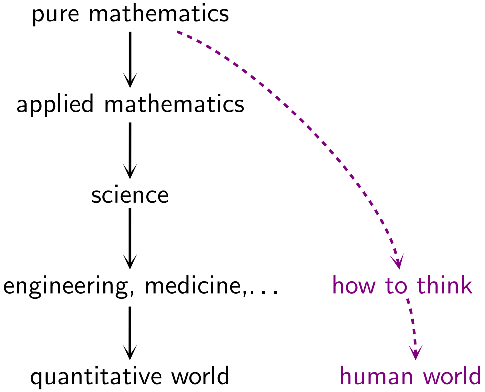
↓

engineering, medicine,. . .

↓

quantitative world

Pure mathematics is a framework for agreeing on things.



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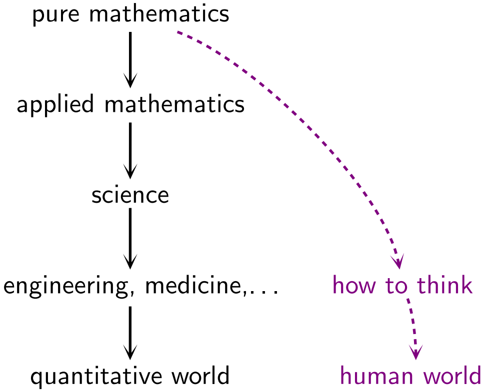

Mathematics is
the study of how things work.

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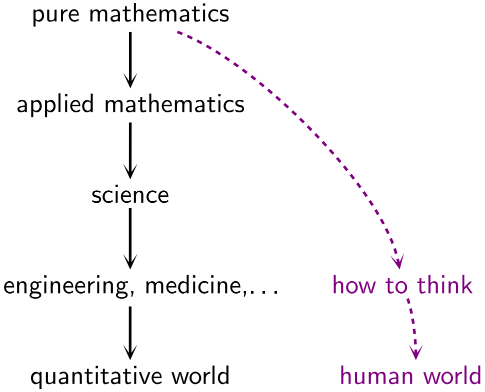

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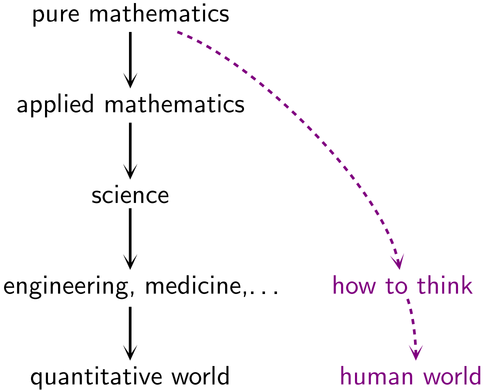

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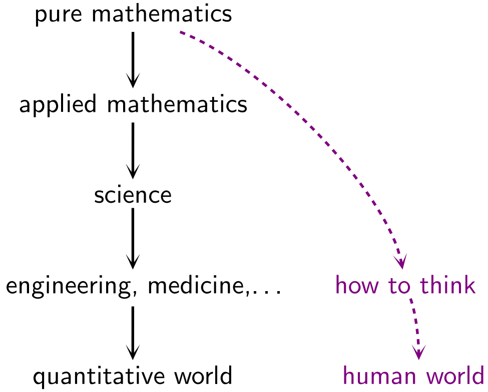

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	logical things	alogical things
logical study	math	
alogical study		

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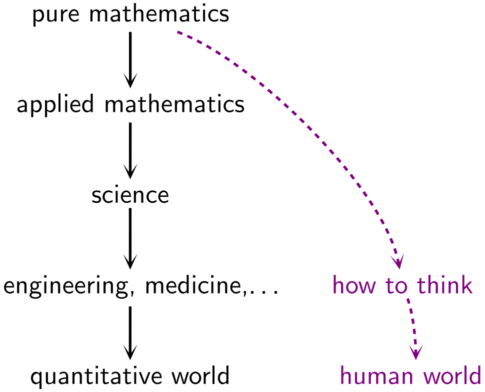

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What other things can you think of?

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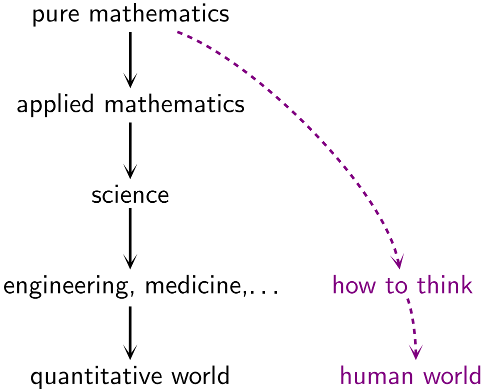

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logical study	math	philosophy
alogical study	numerology	astrology

What other things can you think of?

1. Introduction: About me

Previous teaching:

- math majors at research universities
- all students had done very well in high school math
- careers in math, science, engineering, finance, accountancy, law, teaching
- male dominated ($< 30\%$ female).

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Current teaching:

- art students at art school (liberal arts)
- many students who did not do well in high school math
- careers in art, design, photography, teaching
- female dominated ($> 90\%$ female).

Art students: what put them off math?

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memorization

times tables

formulae

getting things wrong

art/math false dichotomy

lack of creativity

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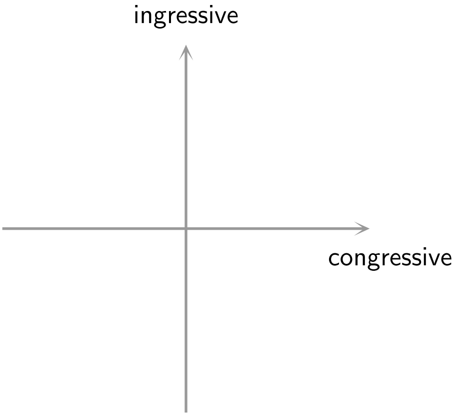

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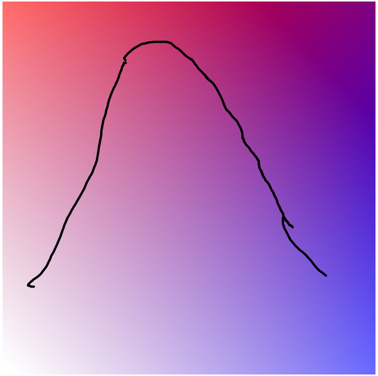

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This is not a classification of people,
but a descriptive characterization of behaviors and interactions.

Ingressive

focusing on oneself
imposing on people
independence and individualism
competitive and adversarial
selective or single-track

Congressive

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focusing on oneself
imposing on people
independence and individualism
competitive and adversarial
selective or single-track

focusing on society and community
taking others into account
interdependence and connectedness
collaborative and cooperative
circumspect

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outcome

facts and rules

solving problems

calculate answers

traditional lecturing

right/wrong

exams and competitions

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outcome	process
facts and rules	investigation, discovery, choices, reasons
solving problems	building structures
calculate answers	uncover relationships
traditional lecturing	project based learning
right/wrong	low floor/high ceiling
exams and competitions	festivals and fairs

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"Now I find that math is not just a tool, and the real reason for learning math is to practice our thinking".

Pure mathematics is a theory of analogies

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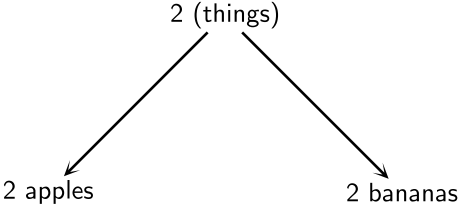
2 apples

2 bananas

Pure mathematics is a theory of analogies



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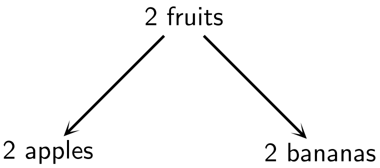

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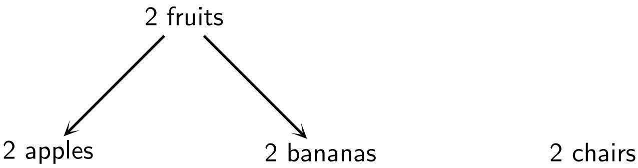

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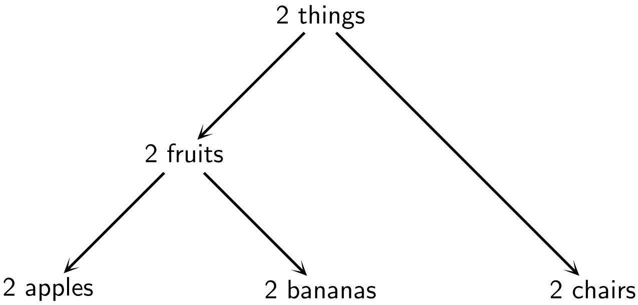

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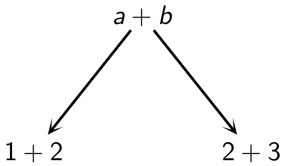

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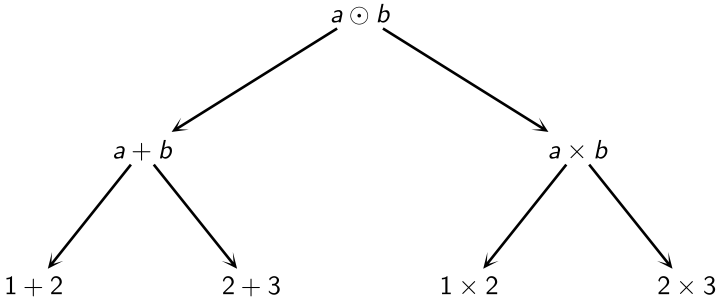

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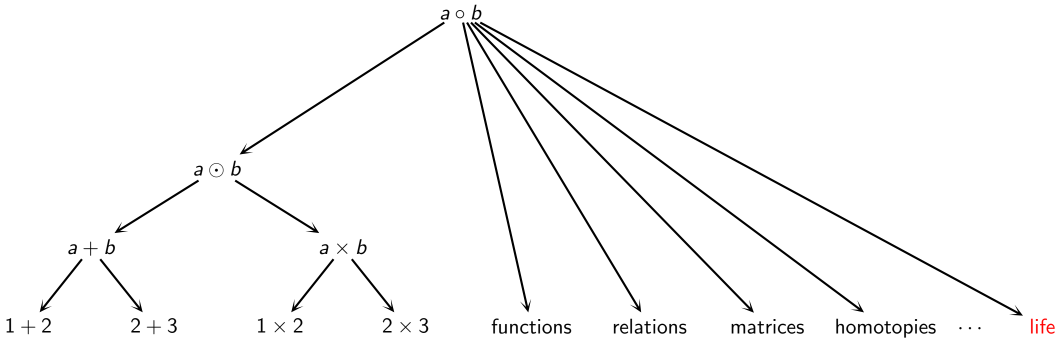

Pure mathematics is a theory of analogies

puff pastry

viruses

2. Abstraction and analogies

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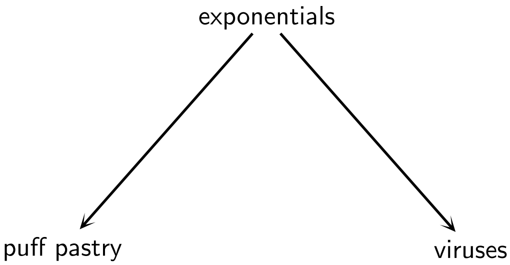

COVID-19

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seasonal flu

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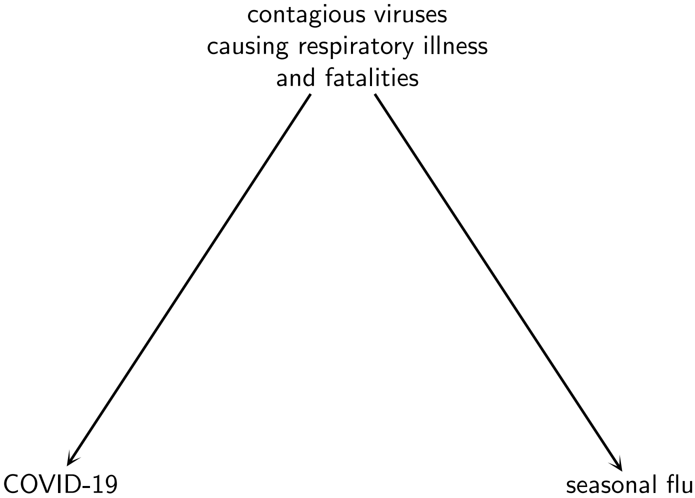

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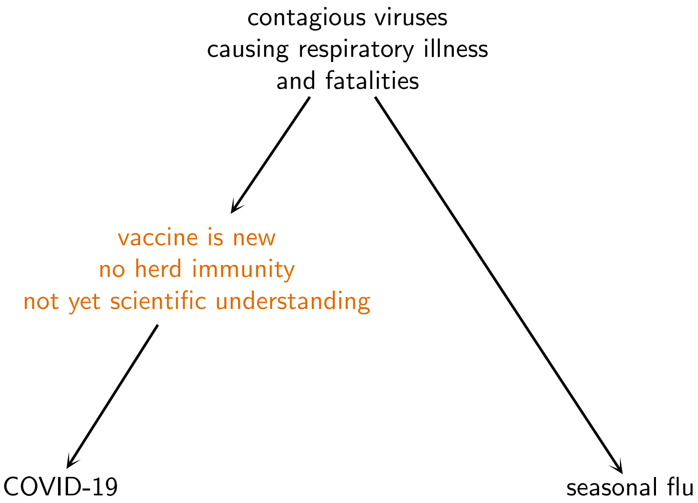

The idea of category theory

- relationships
- structure
- context
- nuanced ways to think about sameness

"A is the same age as B" is either true or false.

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2. For all A and B
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Definition:

If a relation is reflexive, symmetric and transitive it's called an equivalence relation.

Note that these properties only count as true if the condition is true for all objects.

If it sometimes holds and sometimes doesn't, the relation itself does not have the property.

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Explore

i. [is the same height as] R, S, T
ii. is taller than T
iii. is mother of
iv. is a friend of
v. is east of
vi. is in an orchestra with S
vii. $\leq$ R, T
viii. is a factor of R, T
ix. is roughly the same as (first create a definition)

Not many things are equivalence relations

14.

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Further: An equivalence relation is just a partition of the set into subsets with no overlap.

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Categories allow for more general kinds of relation so that we can include more examples like

- equivalence relations
- $\leq$
- a factor of
- functions
- matrices
- symmetry
- paths in a space
- $\vdots$

Break to 6:45

Definition: a category $\mathcal{C}$ consists of:

15.

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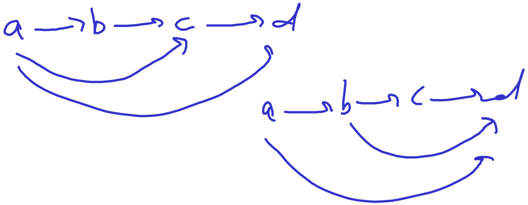
- associativity: given $a \xrightarrow{f} b \xrightarrow{g} c \xrightarrow{h} d$

$$(h \circ g) \circ f = h \circ (g \circ f)$$



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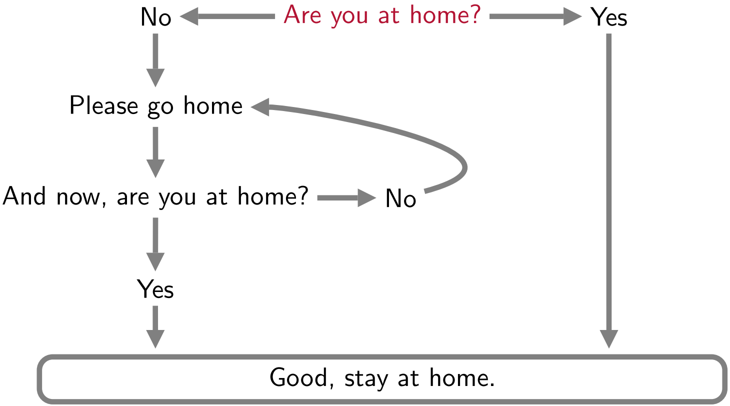
What happened to symmetry?

We don't demand it but we look for it afterwards. This is the notion of "sameness" in a category.

Note: arrows are also called morphisms or maps

In a way composition is the whole point of a category.
It is what makes it more than a flow chart.



Numbers

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- morphisms: $a \longrightarrow b$ whenever $a \leq b$

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Thoughts.

i. Try drawing this category. Remember to omit unnecessary arrows.

ii. Infinity is not in this category. Why? If we put it in, where will it go?

iii. Why do we have to use $\leq$ and not $<$ for this to be a category?

5. Examples: numbers

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- morphisms: $a \longrightarrow b$ whenever $a \leq b$

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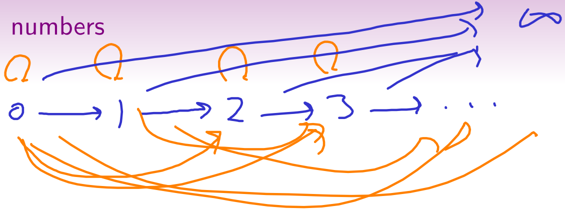

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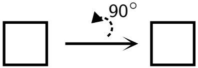
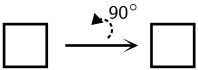
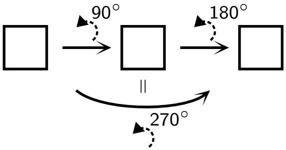
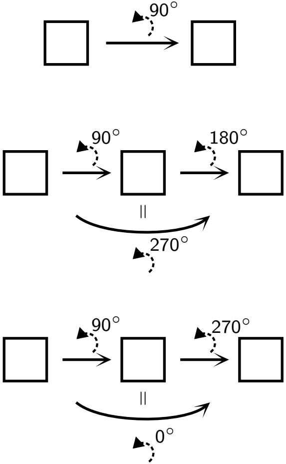


We can regard symmetry as a relation



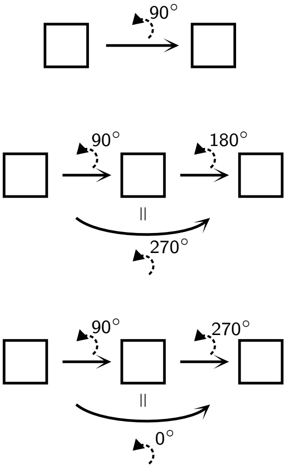


We can regard symmetry as a relation



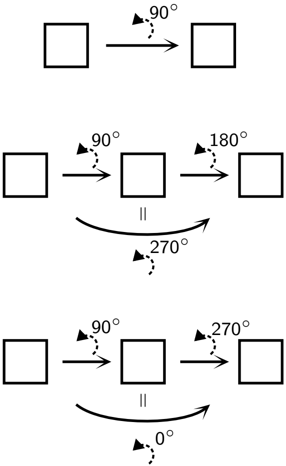
5. Examples: symmetries

We can regard symmetry as a relation



	0	90	180	270
0				
90			270	0
180				
270				

We can regard symmetry as a relation



	0	90	180	270
0	0	90	180	270
90	90	180	270	0
180	180	270	0	90
270	270	0	90	180

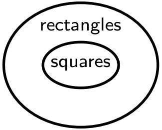
Fill in the rest of the table.

i. Do you see a pattern? Does it remind you of anything? If so, why?

ii. Theorize? Generalize?

Quadrilaterals

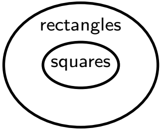
19.



square —special case of→ rectangle

rhombus —special case of→ parallelogram

Quadrilaterals



square $\xrightarrow{\text{special case of}}$ rectangle

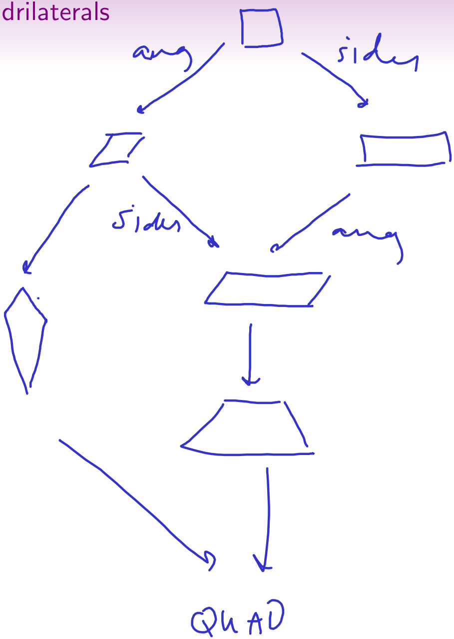
rhombus $\xrightarrow{\text{special case of}}$ parallelogram

Try putting all types of quadrilateral in a diagram. Remember to omit unnecessary arrows.

i. What actual process is happening along each arrow?

ii. Where are there two different paths between the same things?

iii. What are some reasons to draw it like this instead of a Venn diagram?



- objects: factors of 30
- morphisms: $a \longrightarrow b$ whenever a is a multiple of b

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- morphisms: $a \longrightarrow b$ whenever a is a multiple of b

20.

$1,$

6. Examples: Factors and privilege

- objects: factors of 30
- morphisms: $a \longrightarrow b$ whenever a is a multiple of b

$1, \ 2,$

- objects: factors of 30
- morphisms: $a \longrightarrow b$ whenever a is a multiple of b

20.

$1, \; 2, \; 3,$

- objects: factors of 30
- morphisms: $a \longrightarrow b$ whenever a is a multiple of b

$1, \ 2, \ 3, \ 5,$

- objects: factors of 30
- morphisms: $a \longrightarrow b$ whenever a is a multiple of b

$1, \; 2, \; 3, \; 5, \; 6,$

6. Examples: Factors and privilege

- objects: factors of 30
- morphisms: $a \longrightarrow b$ whenever a is a multiple of b

20.

$$1,\ 2,\ 3,\ 5,\ 6,\ 10,$$

- objects: factors of 30
- morphisms: $a \longrightarrow b$ whenever a is a multiple of b

20.

$1,\ 2,\ 3,\ 5,\ 6,\ 10,\ 15,$

- objects: factors of 30
- morphisms: $a \longrightarrow b$ whenever a is a multiple of b

$$1,\ 2,\ 3,\ 5,\ 6,\ 10,\ 15,\ 30$$

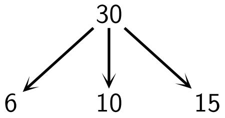
- objects: factors of 30
- morphisms: $a \longrightarrow b$ whenever a is a multiple of b

$1,\ 2,\ 3,\ 5,\ 6,\ 10,\ 15,\ 30$

30

- objects: factors of 30
- morphisms: $a \longrightarrow b$ whenever a is a multiple of b 20.

$$1,\ 2,\ 3,\ 5,\ 6,\ 10,\ 15,\ 30$$



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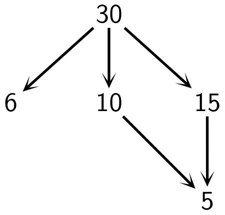
$1,\ 2,\ 3,\ 5,\ 6,\ 10,\ 15,\ 30$



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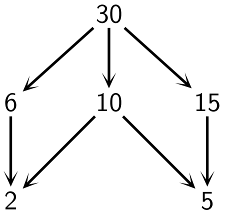

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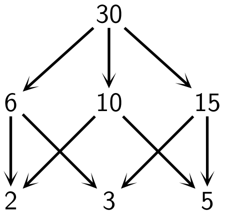

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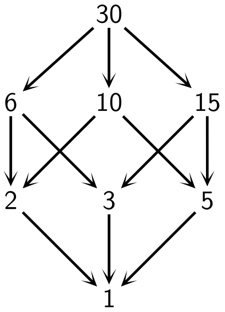

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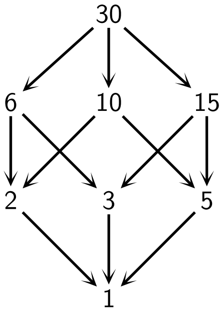

Try some yourself

a) 6
b) 10
c) 8
d) 70
e) 42
f) 12
g) 24
h) hard: 60, 210
i) any number you like

- objects: factors of 30
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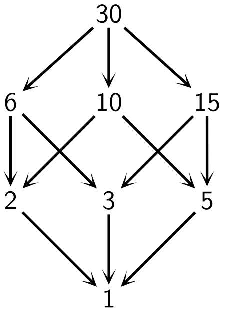
1, 2, 3, 5, 6, 10, 15, 30



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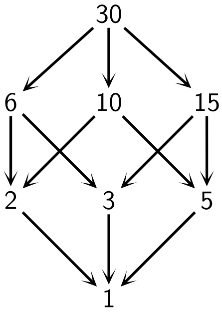
Some things to remember

- Each factor is drawn only once.
- We don't draw arrows that are redundant.

- objects: factors of 30
- morphisms: $a \longrightarrow b$ whenever a is a multiple of b

1, 2, 3, 5, 6, 10, 15, 30



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h) hard: 60, 210
i) any number you like

Some things to remember

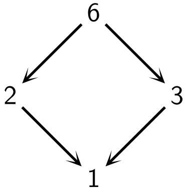
- Each factor is drawn only once.
- We don't draw arrows that are redundant.

Questions

What shapes are produced?
Why do some numbers produce the same shapes?
What is going on and why?

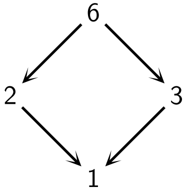
Examples

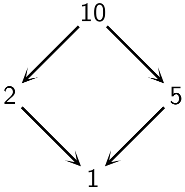
$6: \; 1, 2, 3, 6$



Examples

$6: \; 1, 2, 3, 6$ $\qquad$ $10: \; 1, 2, 5, 10$





6. Examples: Factors and privilege

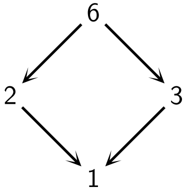
Examples

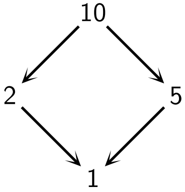
$6: \ 1, 2, 3, 6$

$6 = 2 \times 3$

$10: \ 1, 2, 5, 10$

$10 = 2 \times 5$

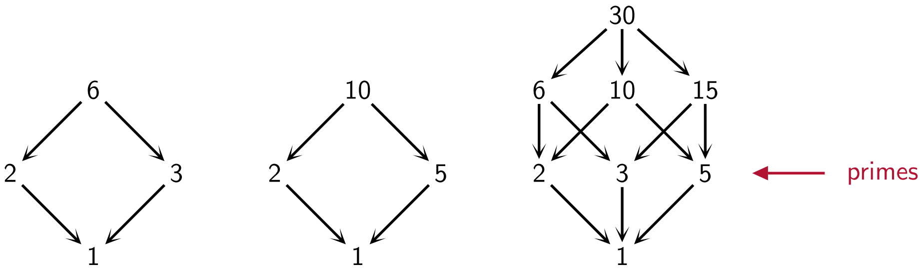




Examples

$6: \ 1, 2, 3, 6$ $\qquad$ $10: \ 1, 2, 5, 10$

$6 = 2 \times 3$ $\qquad$ $10 = 2 \times 5$ $\qquad$ $30 = 2 \times 3 \times 5$



6. Examples: Factors and privilege

Examples

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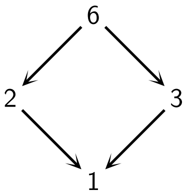

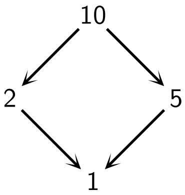

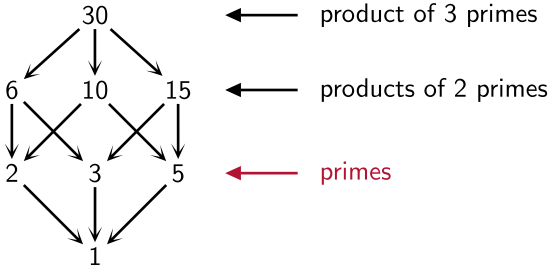

Examples

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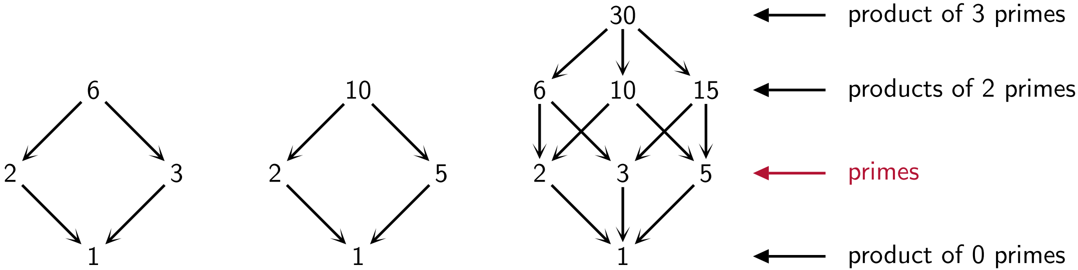

Examples

$12 = 2 \times 2 \times 3$

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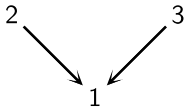
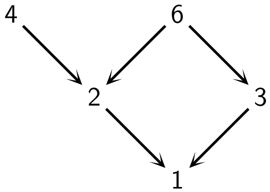


Examples

$12 = 2 \times 2 \times 3$



Examples

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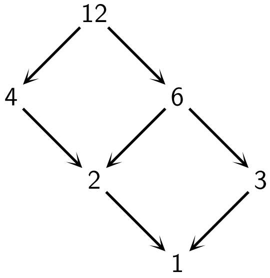
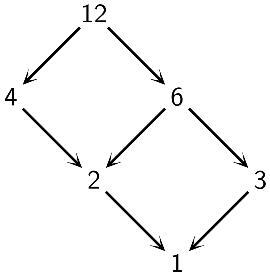


Examples

$12 = 2 \times 2 \times 3$

$24 = 2 \times 2 \times 2 \times 3$



Examples

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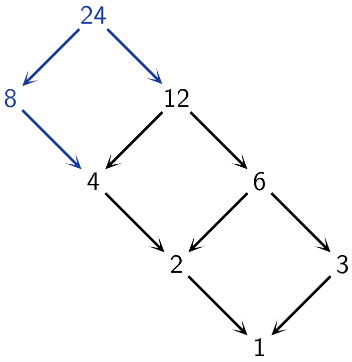

Examples

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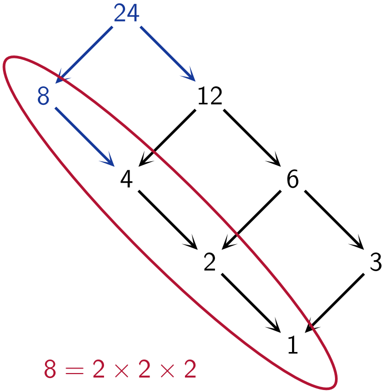


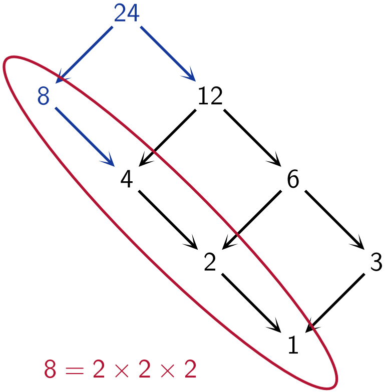
$8 = 2 \times 2 \times 2$

Examples

$12 = 2 \times 2 \times 3$

$36 = 2 \times 2 \times 3 \times 3$

$24 = 2 \times 2 \times 2 \times 3$



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Examples

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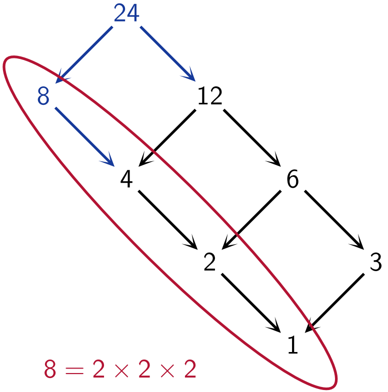
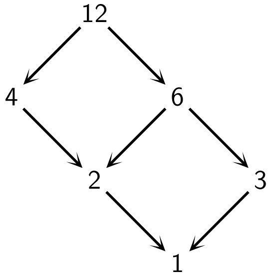
$36 = 2 \times 2 \times 3 \times 3$





Examples

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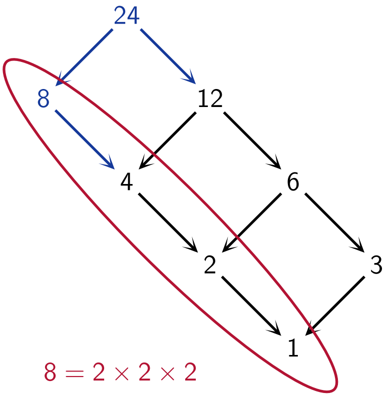

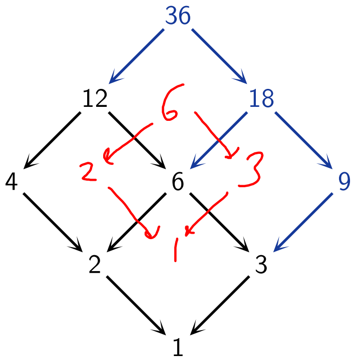

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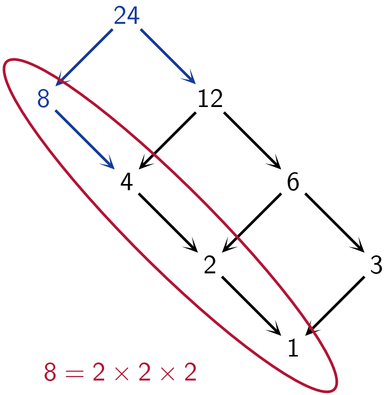

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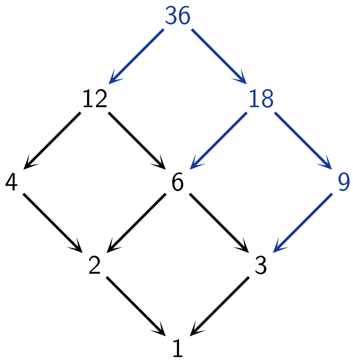


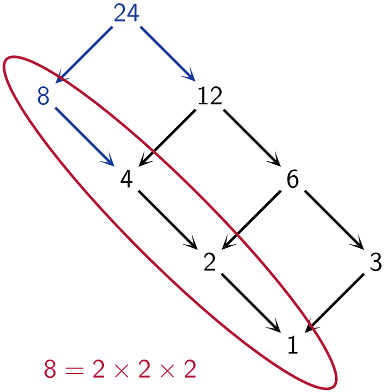
- Number of dimensions $=$ number of distinct prime factors

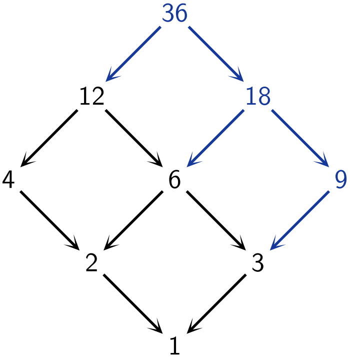
Examples

$12 = 2 \times 2 \times 3$

$24 = 2 \times 2 \times 2 \times 3$

$36 = 2 \times 2 \times 3 \times 3$





- Number of dimensions = number of distinct prime factors
- Lengths of paths = number of repetitions of prime factors

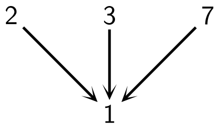
Factors of 42

Factors of 42

1, 2, 3, 6, 7, 14, 21, 42

Factors of 42

1, 2, 3, 6, 7, 14, 21, 42



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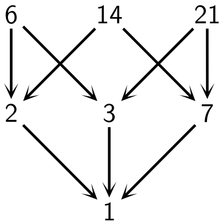

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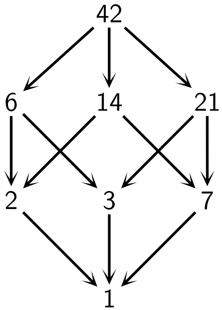

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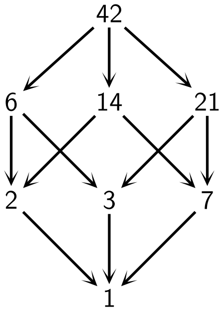

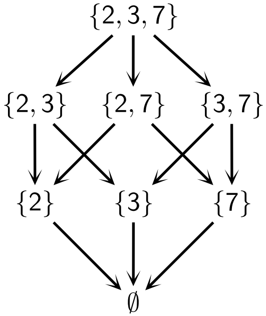

Factors of 42

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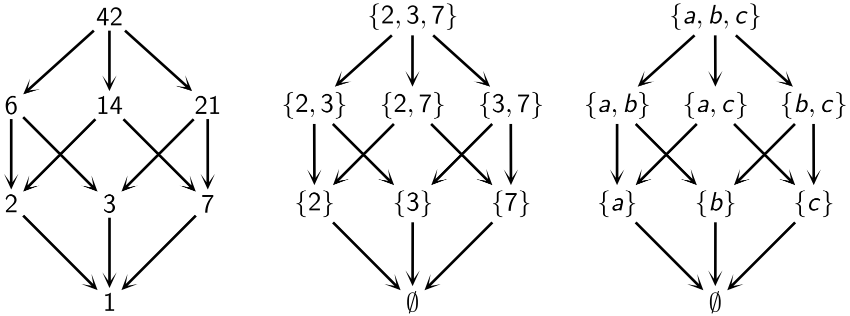

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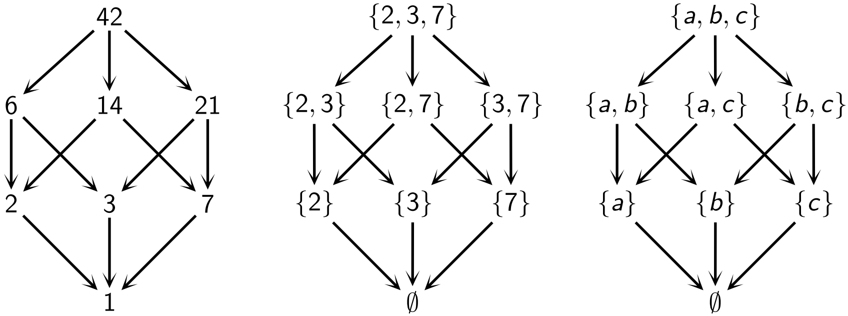
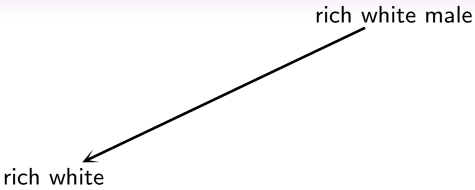


$6 < 7$

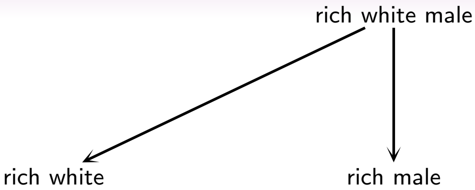
6. Examples: Factors and privilege

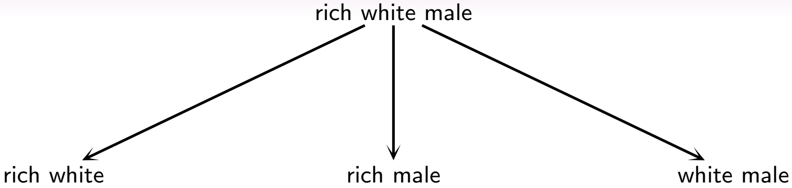
rich white male

6. Examples: Factors and privilege

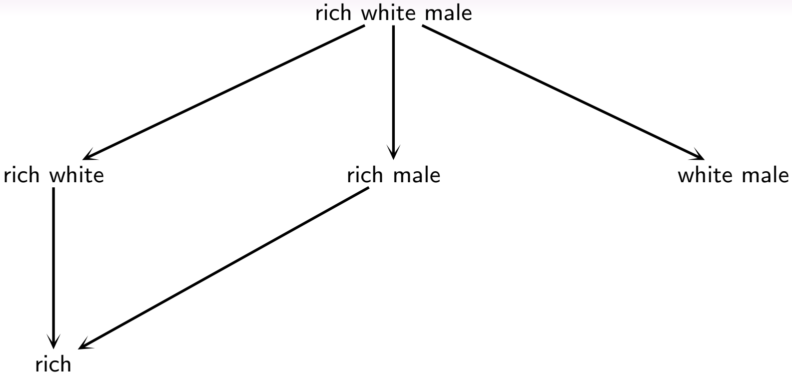


6. Examples: Factors and privilege

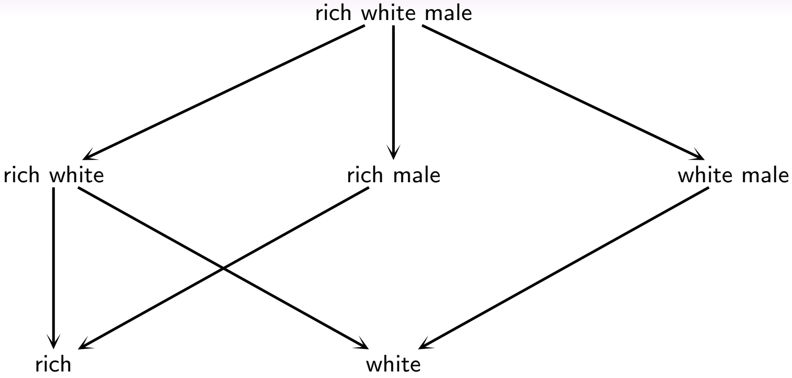



rich white male
rich white
rich male
white male

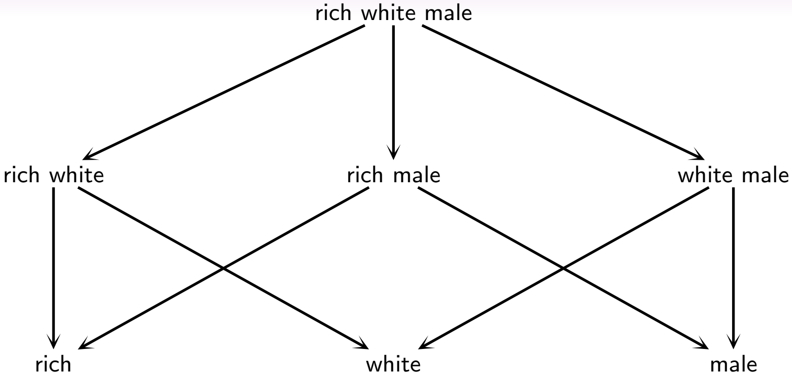
6. Examples: Factors and privilege



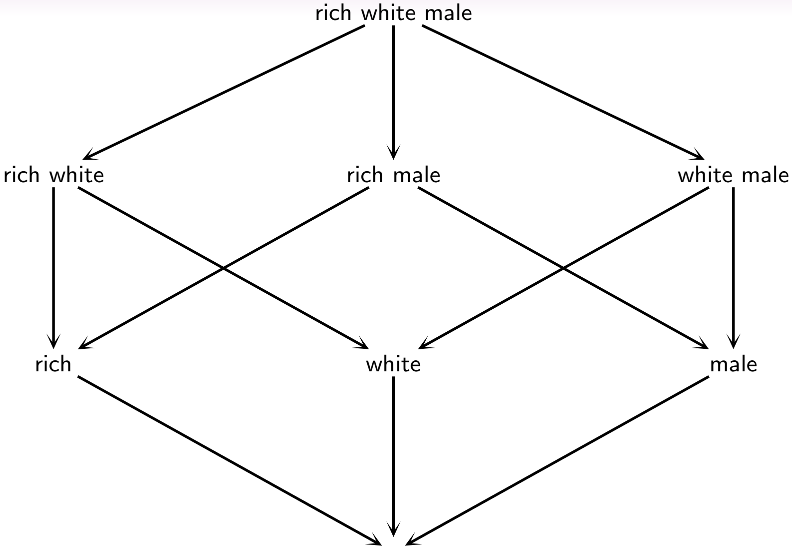
6. Examples: Factors and privilege

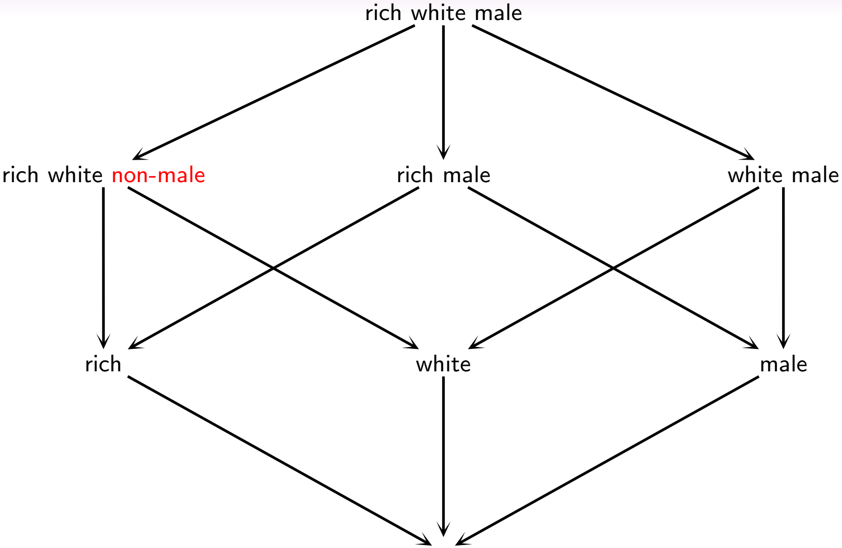


6. Examples: Factors and privilege

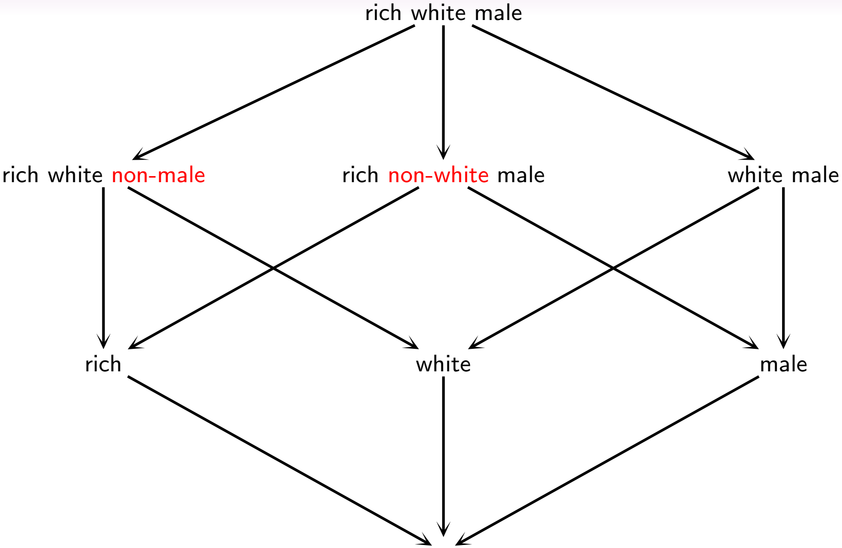


6. Examples: Factors and privilege

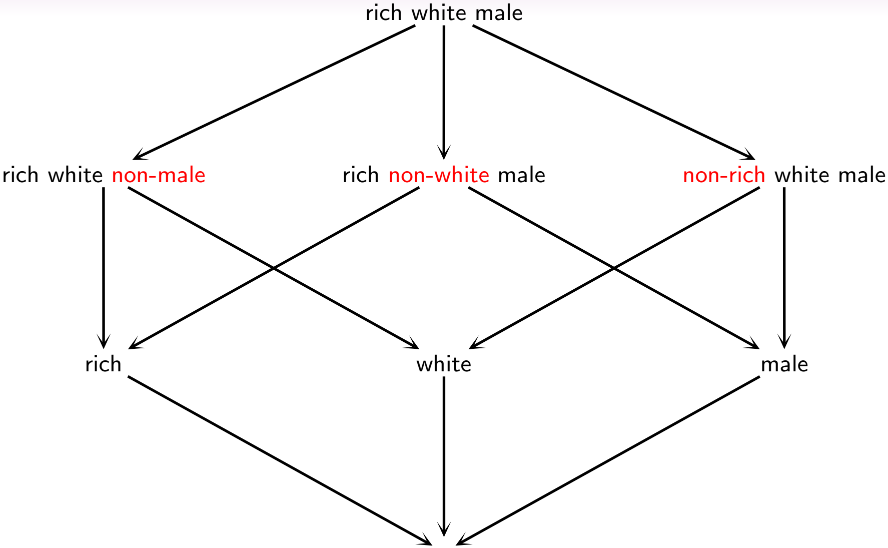



rich white male
rich white non-male
rich male
white male
rich
white
male

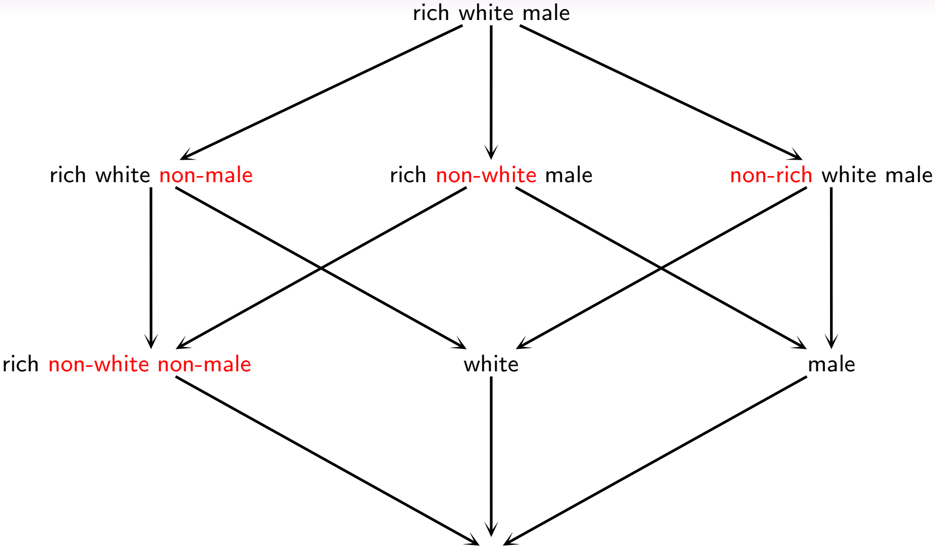
6. Examples: Factors and privilege



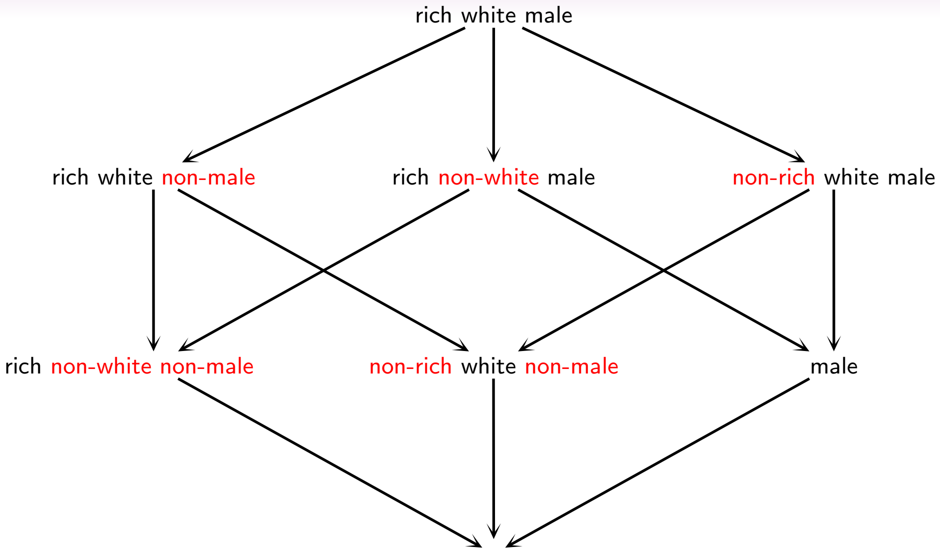
6. Examples: Factors and privilege



6. Examples: Factors and privilege



6. Examples: Factors and privilege



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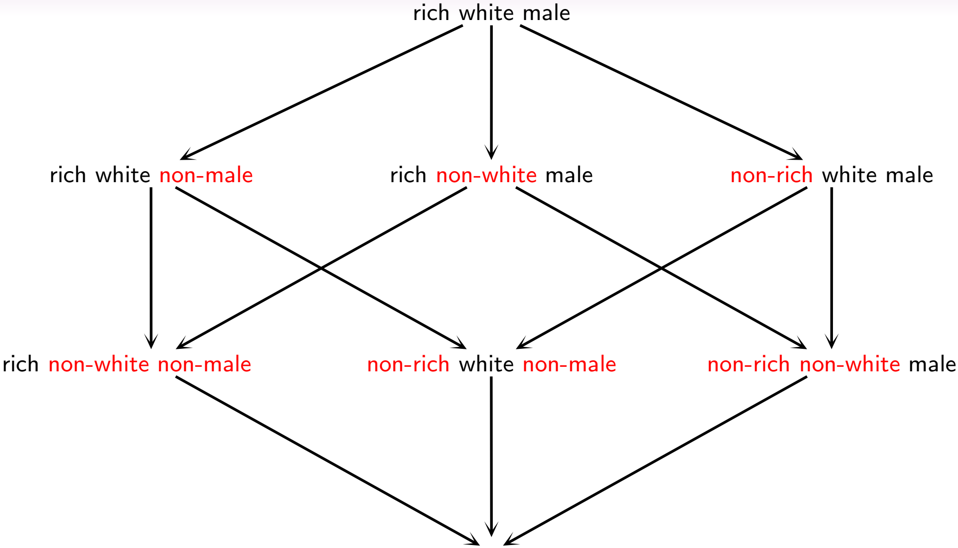

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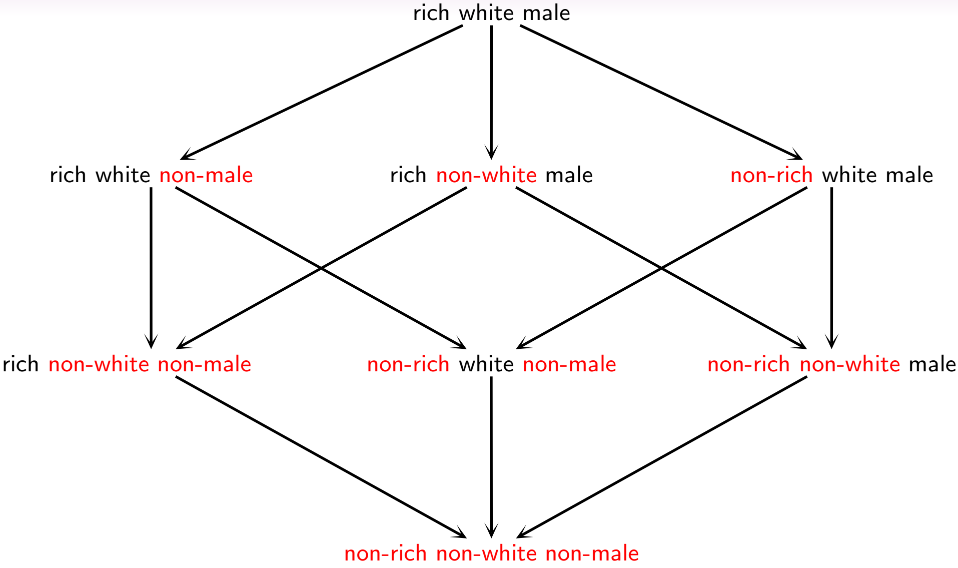
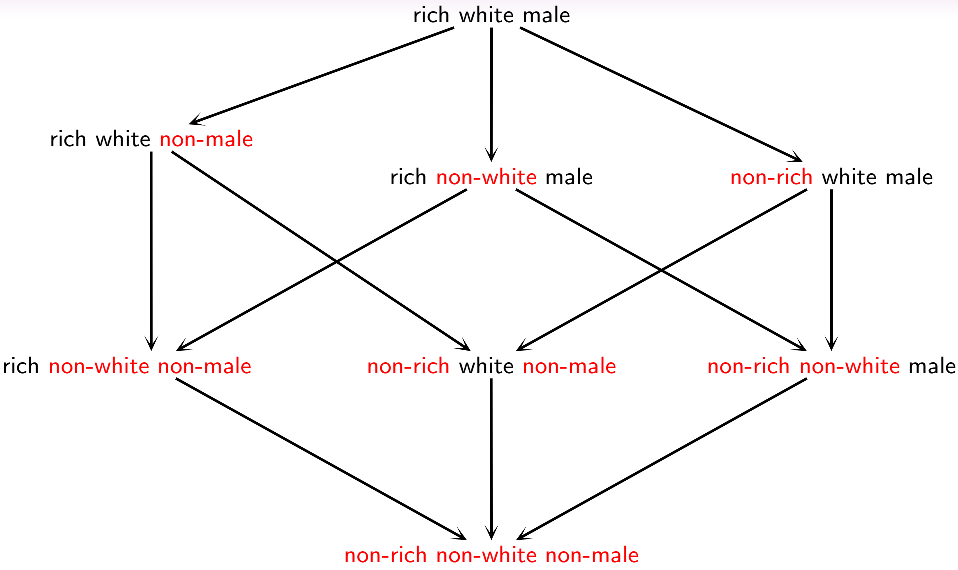
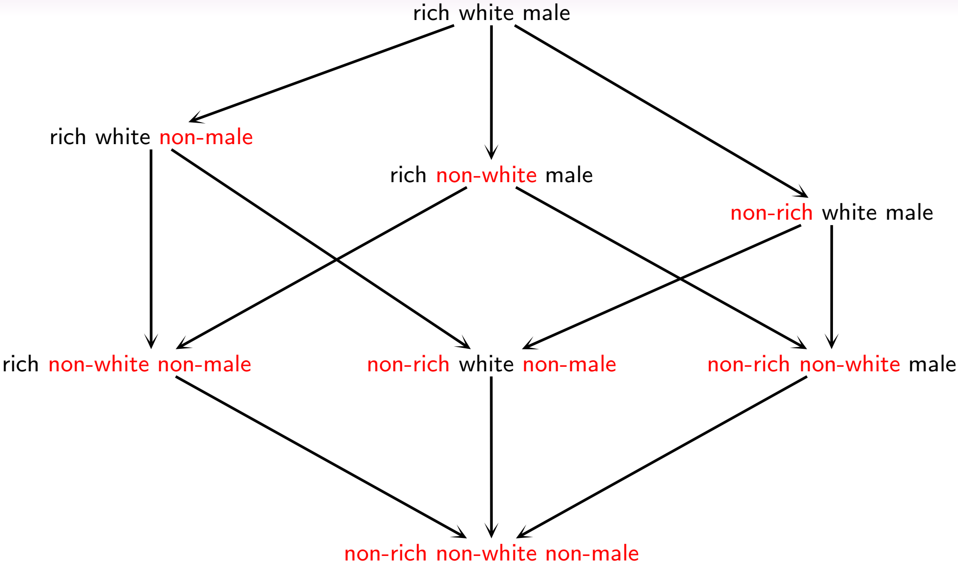



rich white male
rich white non-male
rich non-white male
non-rich white male
rich non-white non-male
non-rich white non-male
non-rich non-white male
non-rich non-white non-male


rich white male
rich white non-male
rich non-white male
non-rich white male
rich non-white non-male
non-rich white non-male
non-rich non-white male
non-rich non-white non-male

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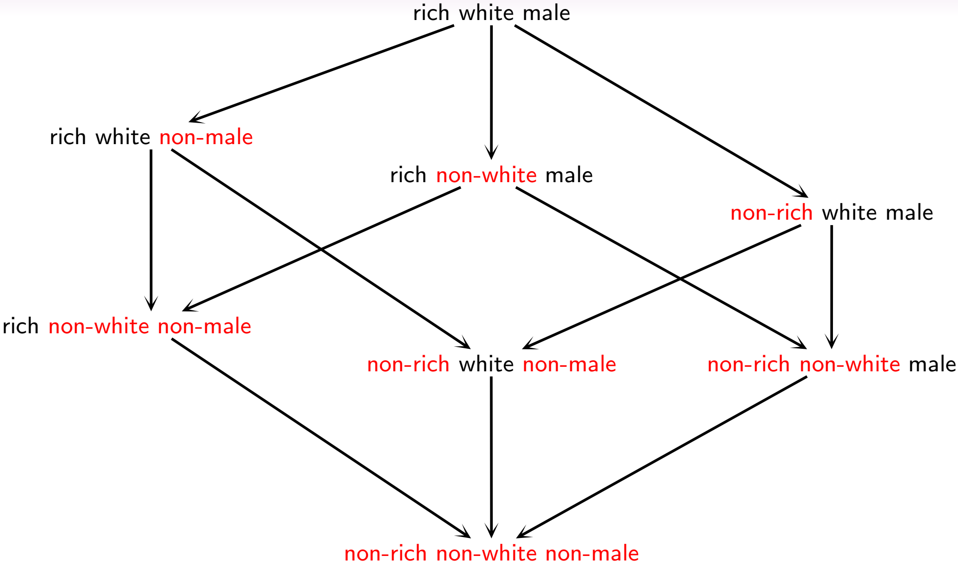

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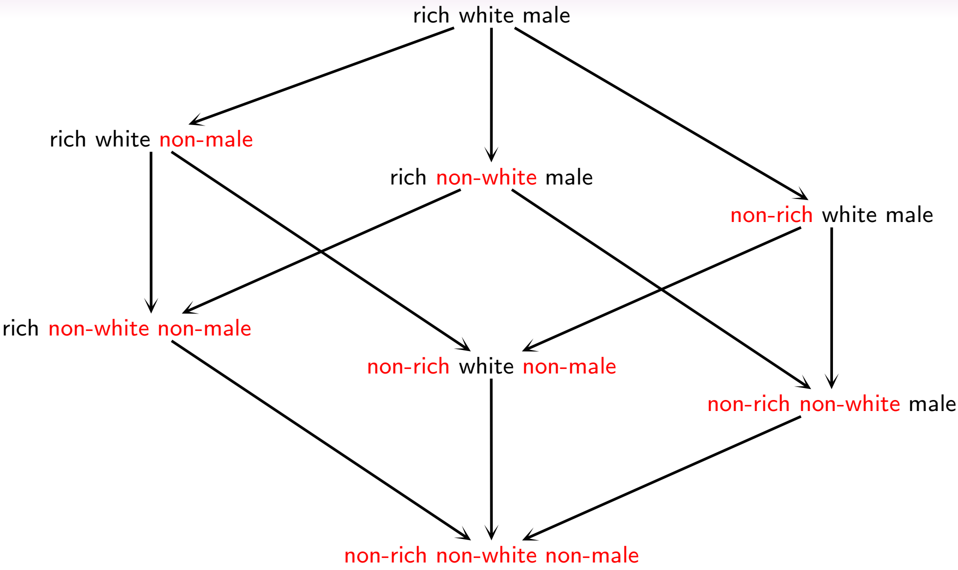

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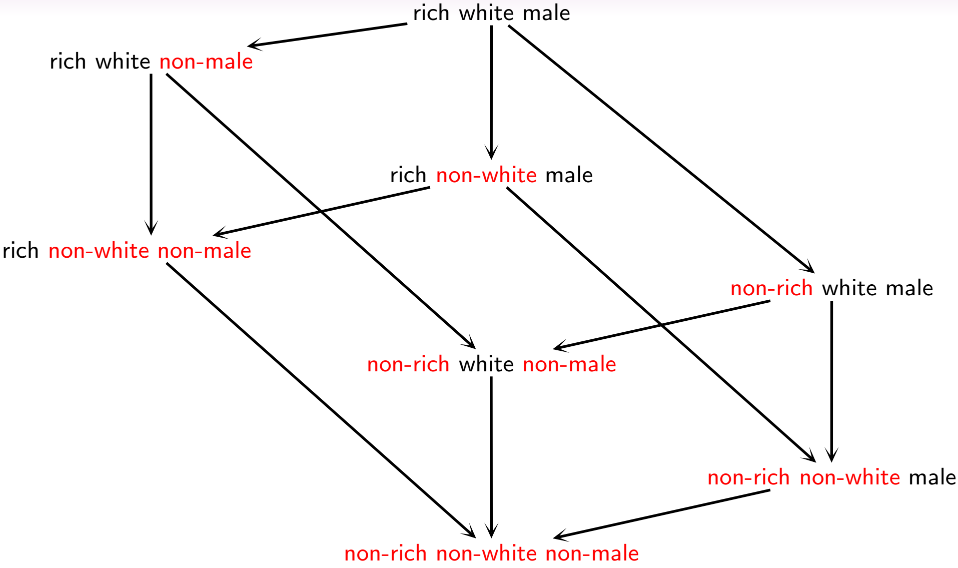

Conclusion

Abstraction has a point.

We gain:

- broader perspective,
- deeper understanding,
- wider applicability, and
- more connections between things.