Privilege structures and generalised metric spaces

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Plan

1. The cuboid of privilege
2. Posets
3. Generalised metric spaces
4. Other examples

1. The cuboid of privilege

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Factors of 30

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1, 2, 3, 5, 6, 10, 15, 30

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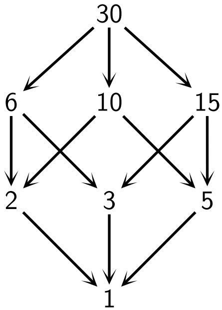

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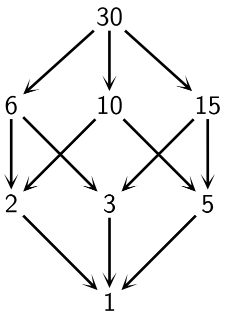

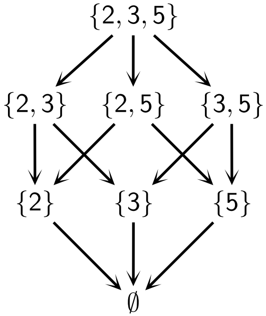

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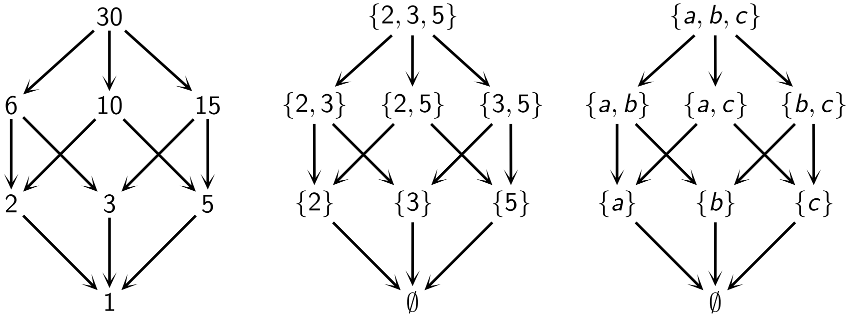

Factors of 42

1, 2, 3, 6, 7, 14, 21, 42

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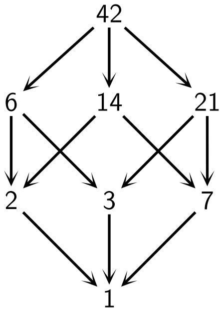
1, 2, 3, 6, 7, 14, 21, 42



Factors of 42

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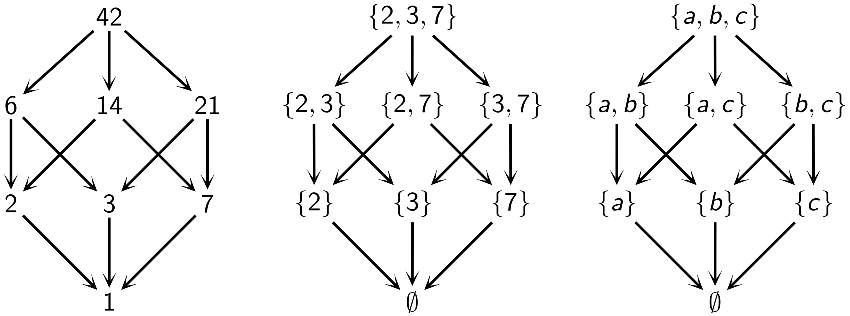

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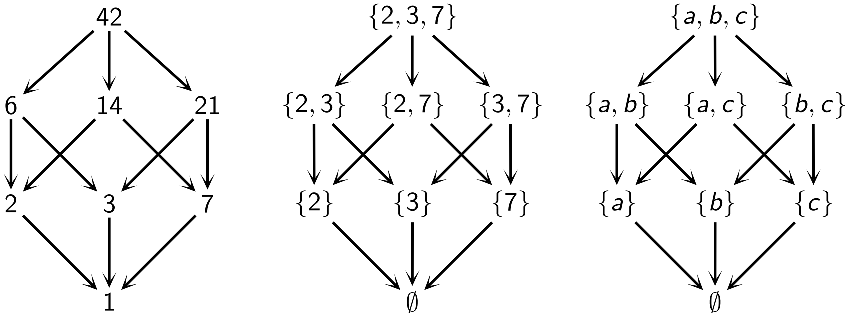

$6 < 7$

1. The cuboid of privilege

rich white male

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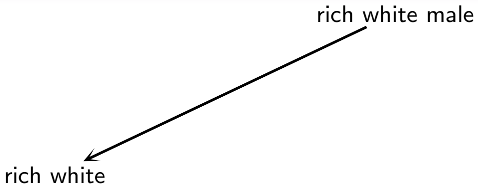

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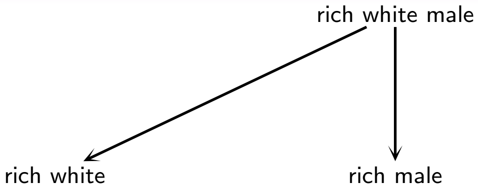

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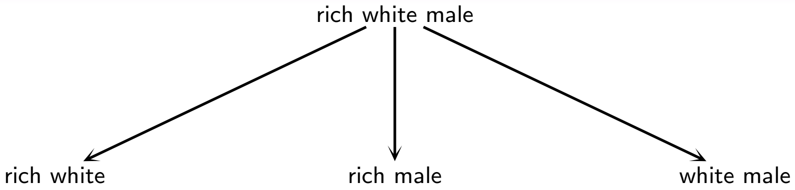

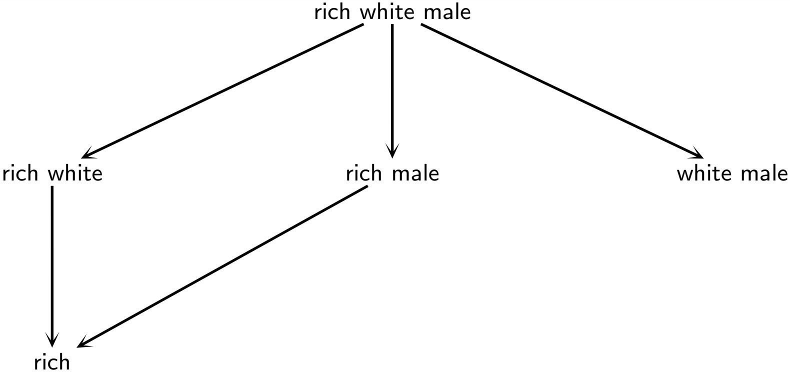
rich white male
rich white
rich male
white male
rich

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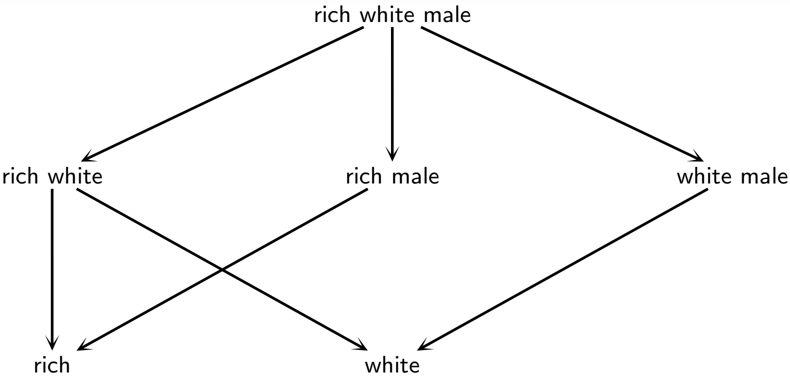

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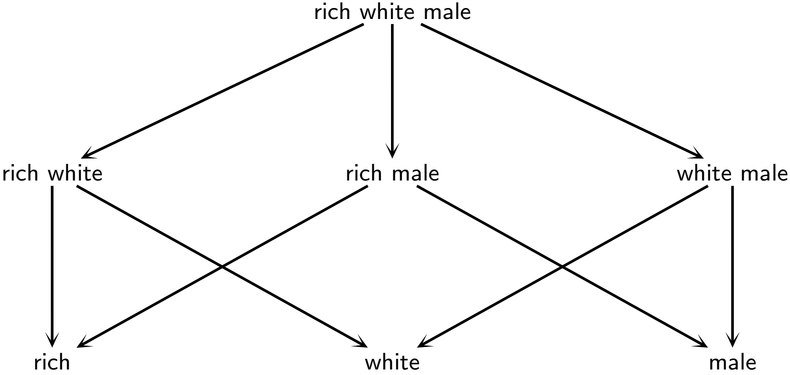

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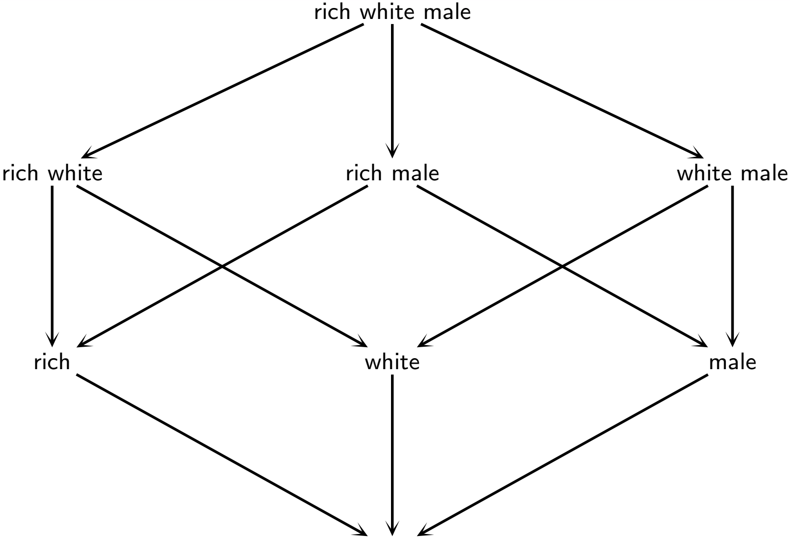

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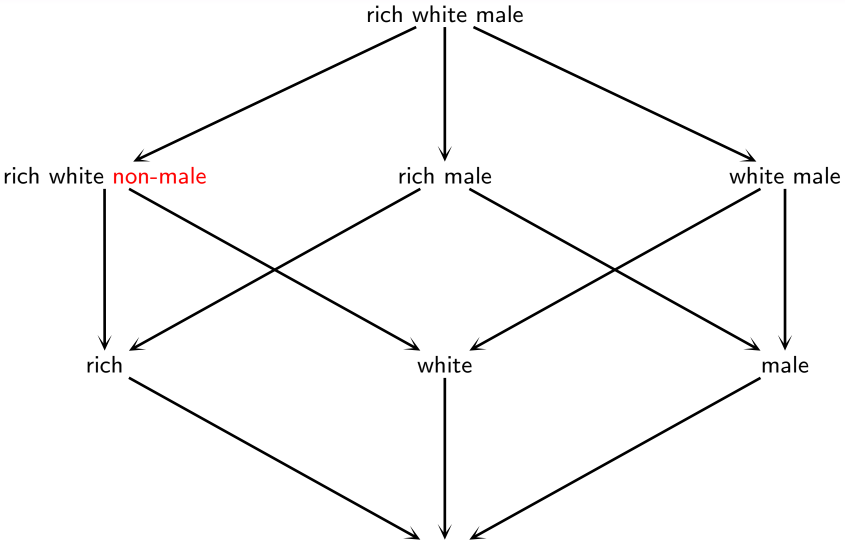

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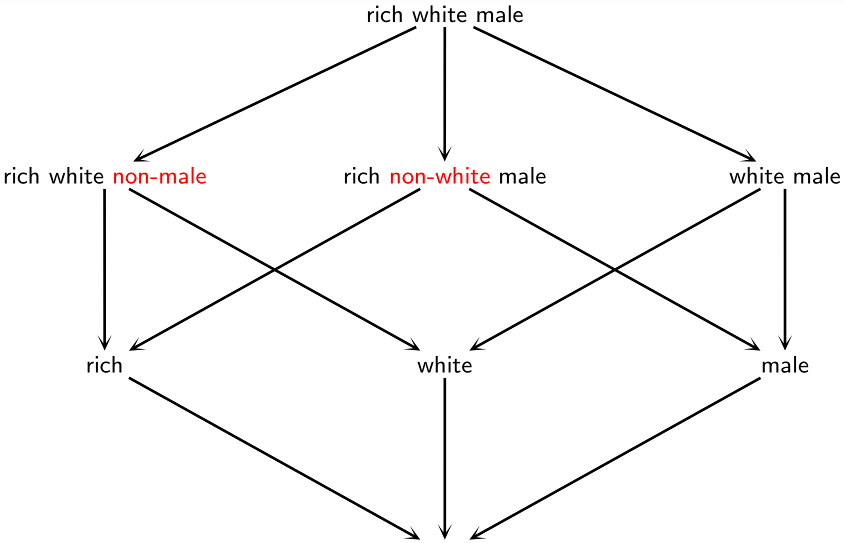

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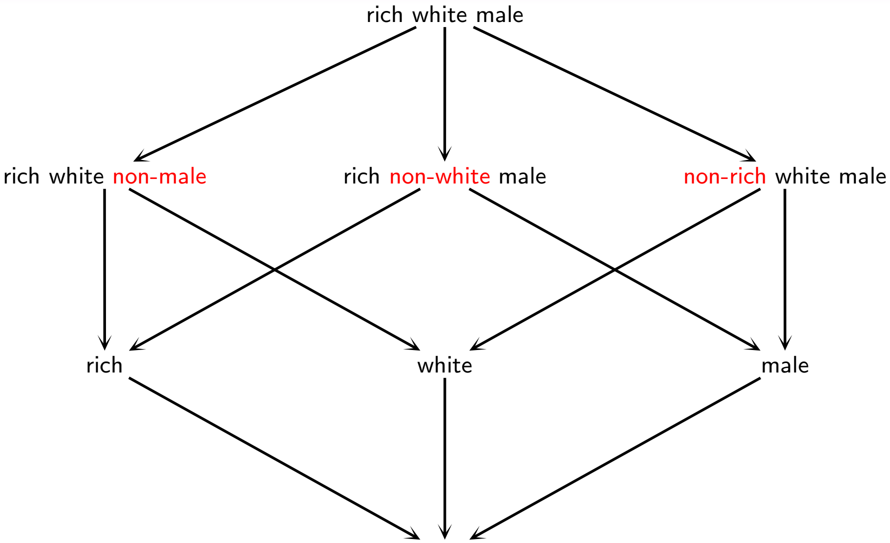

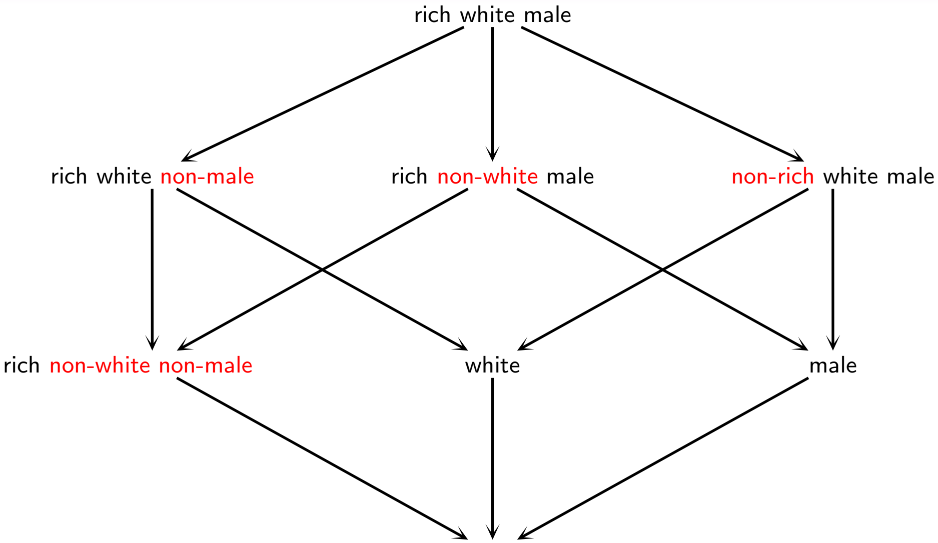
rich white male
rich white non-male
rich non-white male
non-rich white male
rich non-white non-male
white
male

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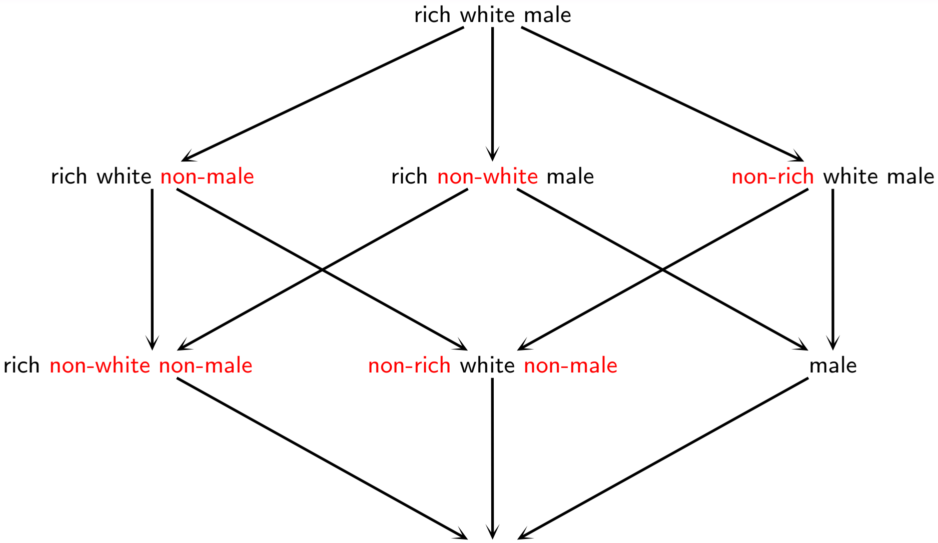

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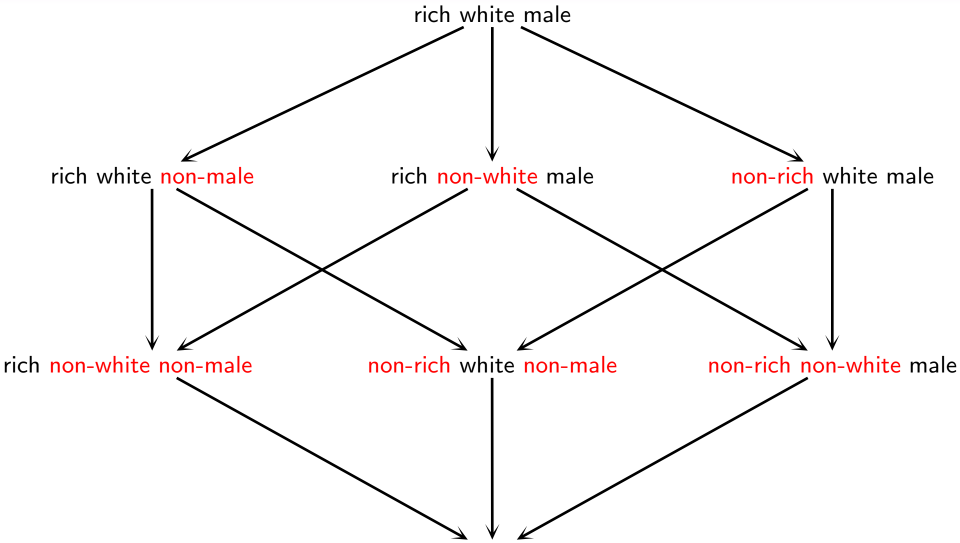

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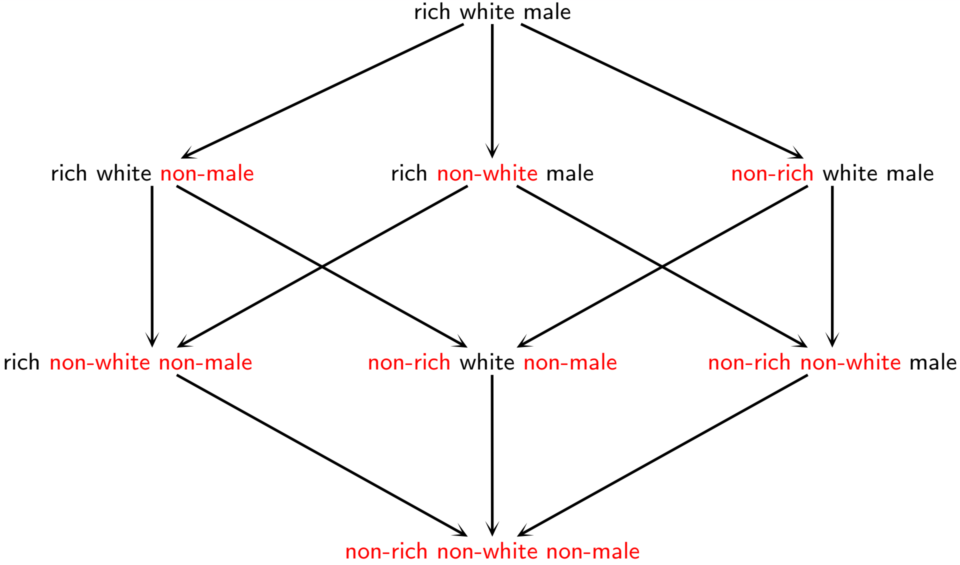

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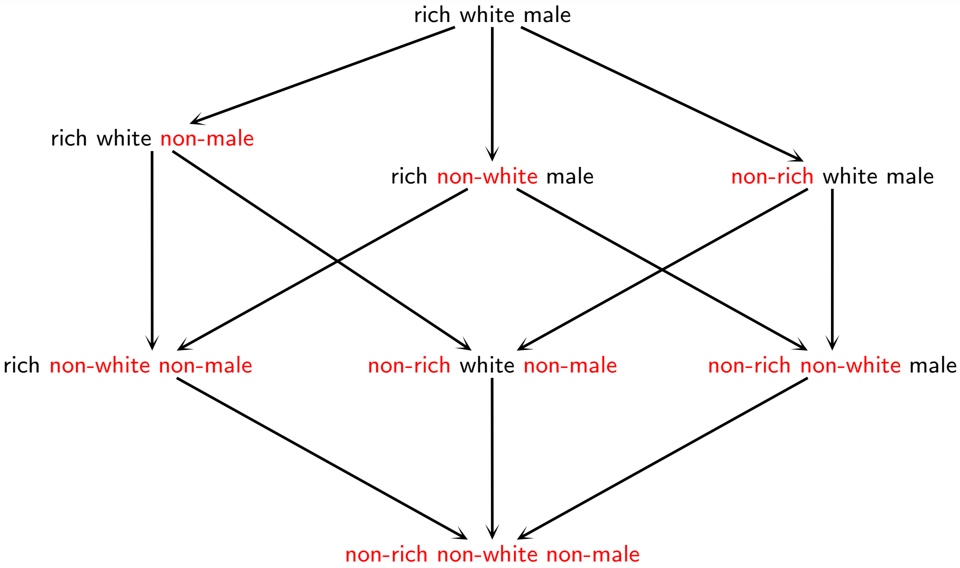

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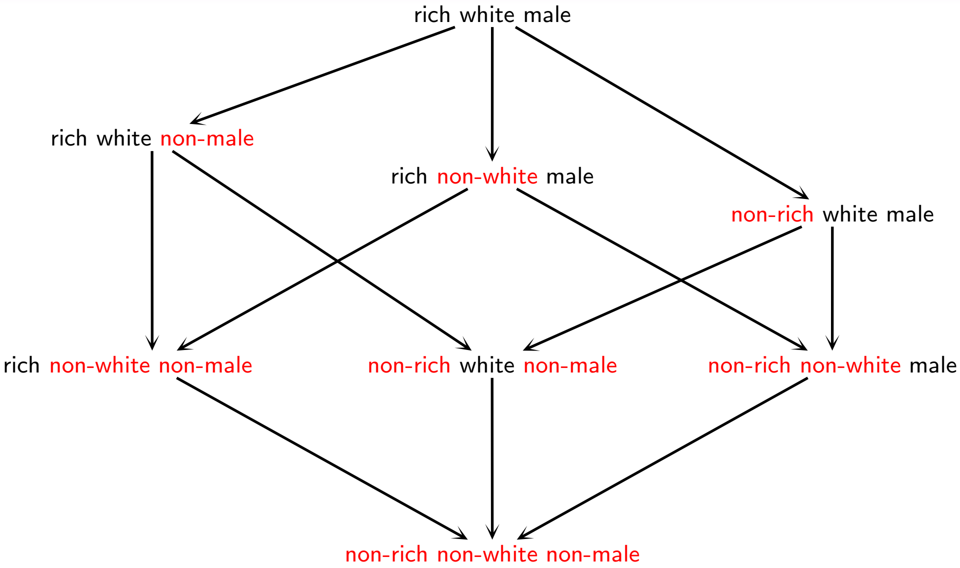

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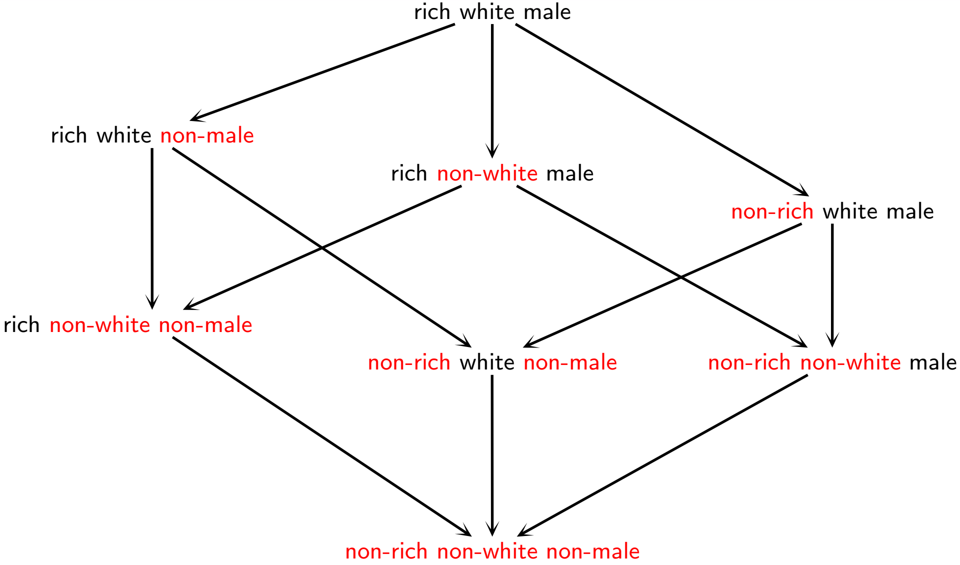

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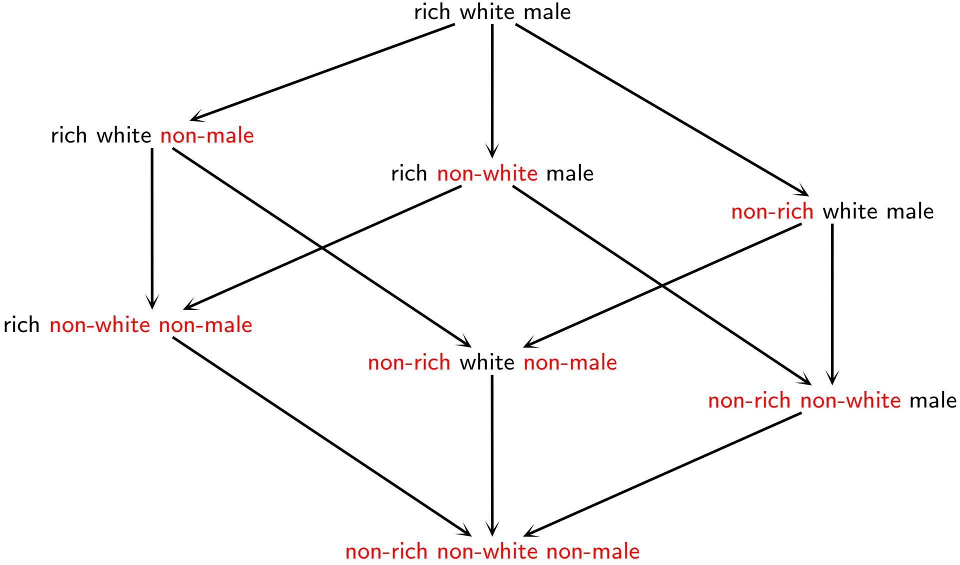

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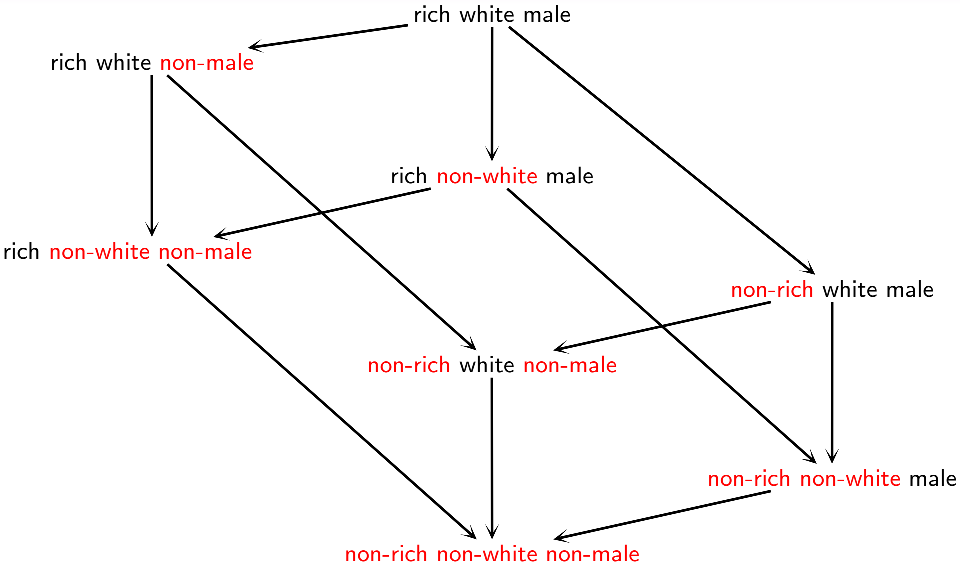

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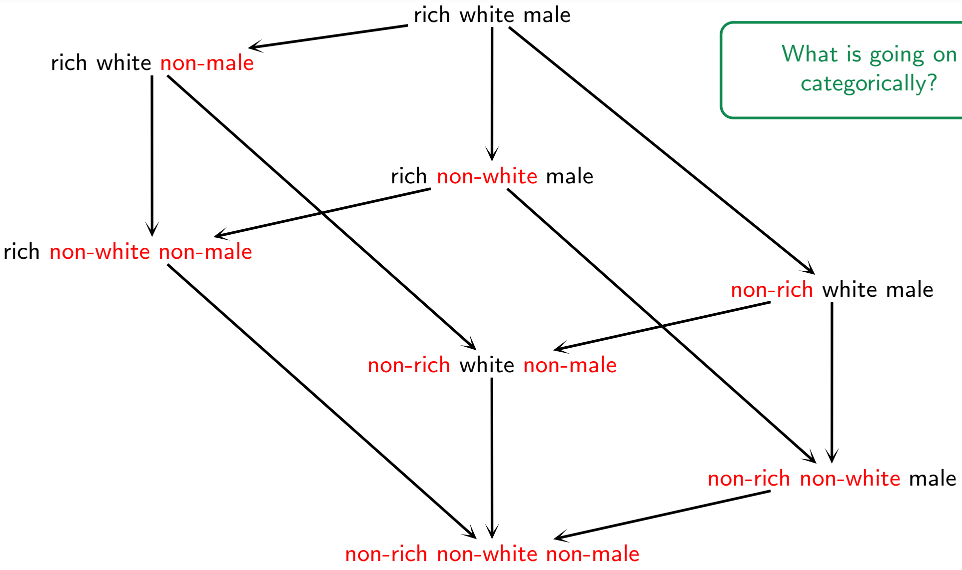
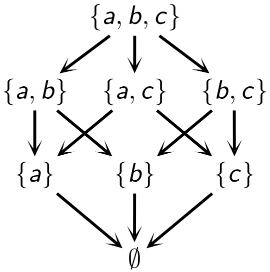


What is going on categorically?

2. Posets: partially ordered sets

A poset is a category with at most one arrow between any two objects



- Morphisms are "assertions".
- For numbers with distinct prime factors, we get the power set poset.
- For numbers with repeated factors...

6.

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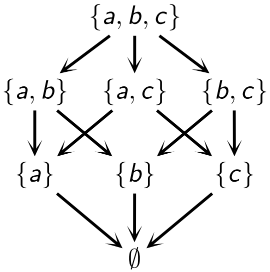

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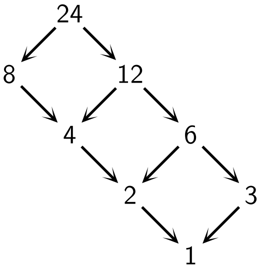

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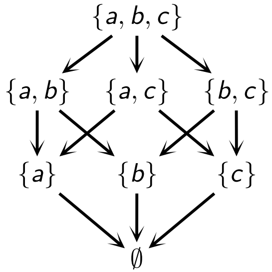

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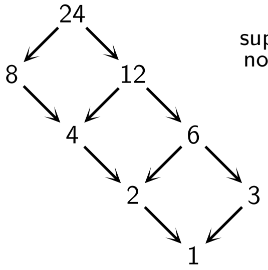

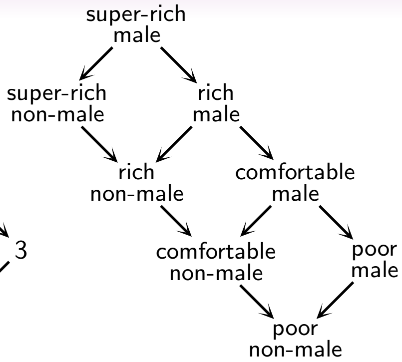

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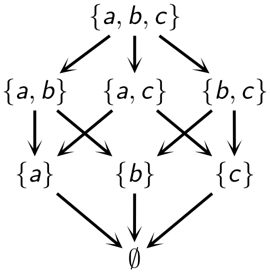

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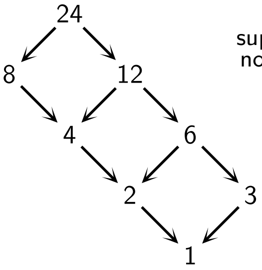
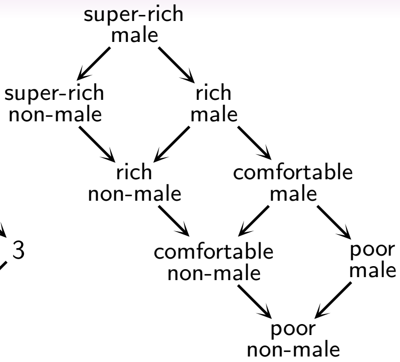


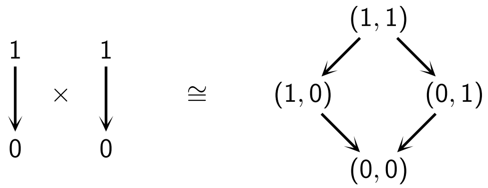


- A toset (totally ordered set) has exactly one morphism between any two objects.

$$0 \longrightarrow 1 \longrightarrow 2 \longrightarrow 3 \longrightarrow \cdots$$

- By contrast a poset can have incomparable elements.
- Functors between posets are order-preserving functions

The category of posets has products
but the category of tosets does not



There is no canonical way to put a total order on the product.

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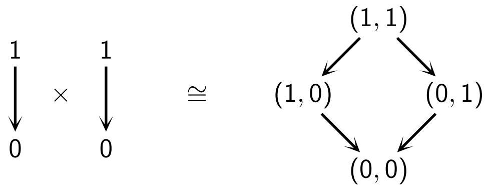

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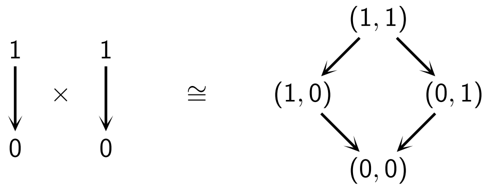

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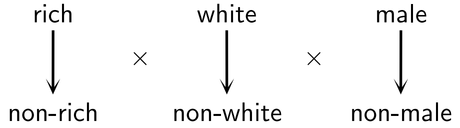
Write I for the directed interval poset $1 \longrightarrow 0$.

Then the cube of privilege "is" I^3.



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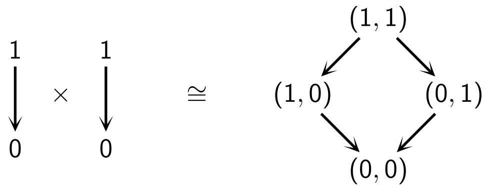

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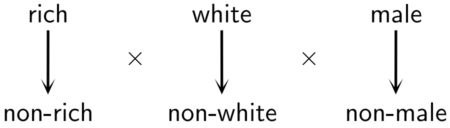
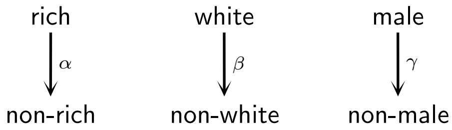


- We can pick any *n* types of privilege
- We can restrict context and consider types of privilege there eg women: rich, white, cis.

Next: incorporate more nuance

3. Generalised metric spaces

Idea: we want to weight the different privileges with real numbers



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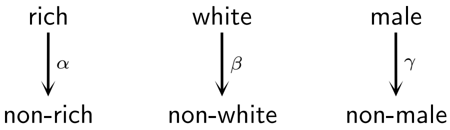

- Instead of morphisms being true/false assertions they will now be real numbers.
- We enrich our category in $\mathbb{R}_{\geq 0}$.
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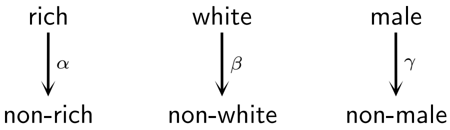

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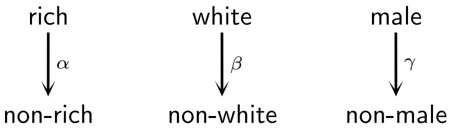

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Definition (Lawvere)

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- objects: $\mathbb{R}_{\geq 0} \cup \{\infty\}$
- morphisms: $x \longrightarrow y$ whenever $x \geq y$
- $\otimes$ is $+$

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$\text{rich} \xrightarrow{\alpha} \text{non-rich}$

$\text{white} \xrightarrow{\beta} \text{non-white}$

$\text{male} \xrightarrow{\gamma} \text{non-male}$

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$[0, \infty]$-categories are generalised metric spaces.

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- For all $a, b \in A_0$, $A(a, b) \in [0, \infty]$

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$[0, \infty]$-categories are generalised metric spaces.

- A set of objects A_0.
- For all $a, b \in A_0$, $A(a, b) \in [0, \infty]$
- Composition: $A(b, c) + A(a, b) \xrightarrow{\geq} A(a, c)$
- Identities: $0 \xrightarrow{\geq} A(a, a)$

Crucial: no symmetry requirement.
Privilege is not symmetric.

The $\otimes$ on $[0, \infty]$ is symmetric
so we can take $\otimes$ of $[0, \infty]$-categories.
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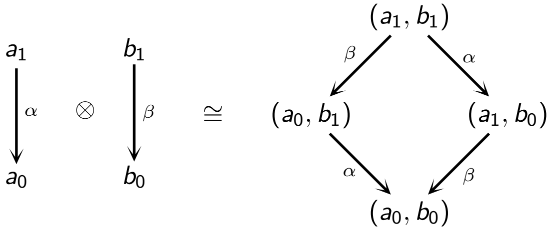

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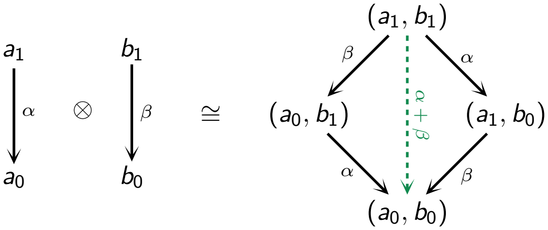

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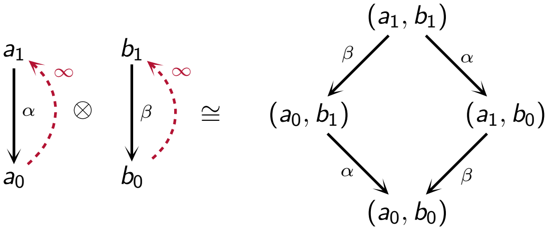

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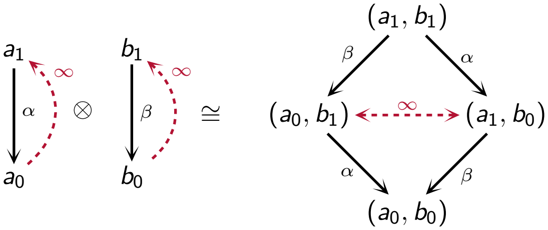

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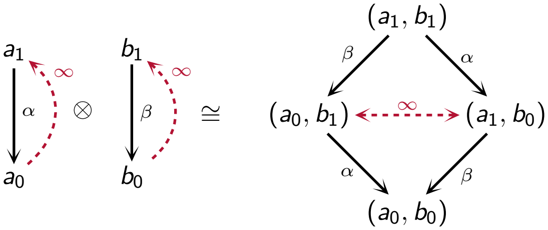

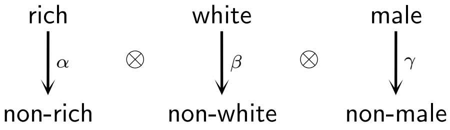

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Remember $\otimes$ is $+$.

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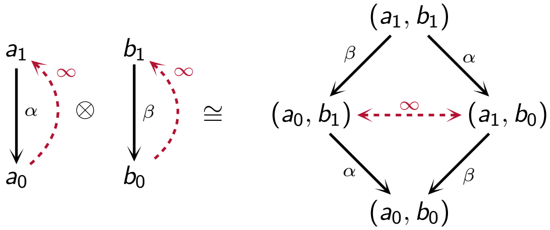

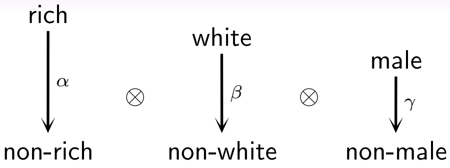

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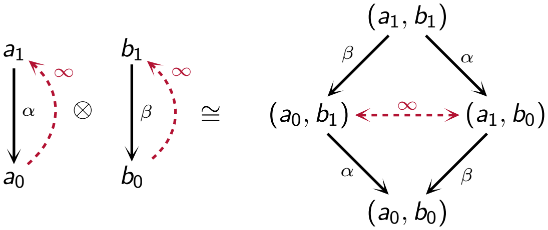

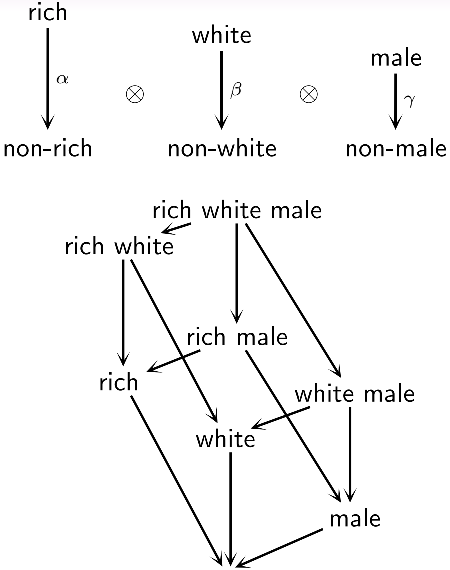

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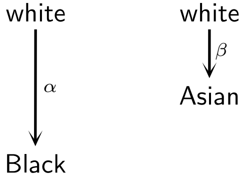

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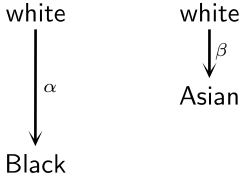

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Posets are just enriched in truth values

- Write $\mathbb{2}$ for the category $\bot \longrightarrow \top$.
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Precisely: $\mathbb{2}$-categories are preorders

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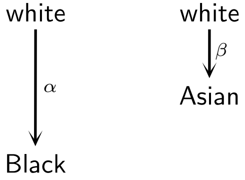

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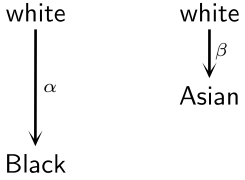

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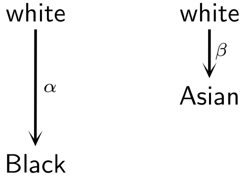

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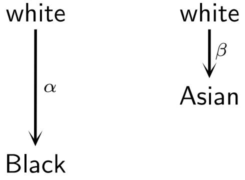

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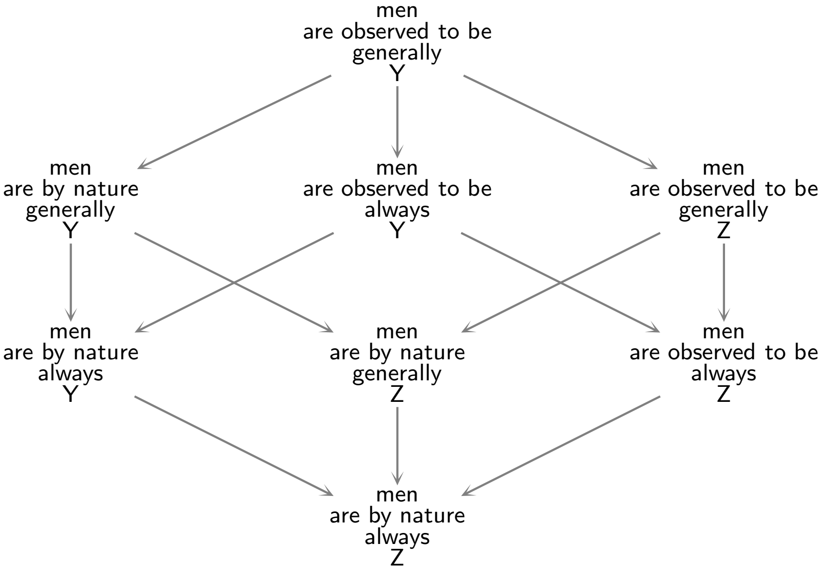
In life we tend to apply F too much, turning a continuum of gray into black-and-white

4. Other examples: fallacious arguments


men are observed to be generally Y
men are by nature generally Y
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men are by nature generally Z
men are observed to be always Z
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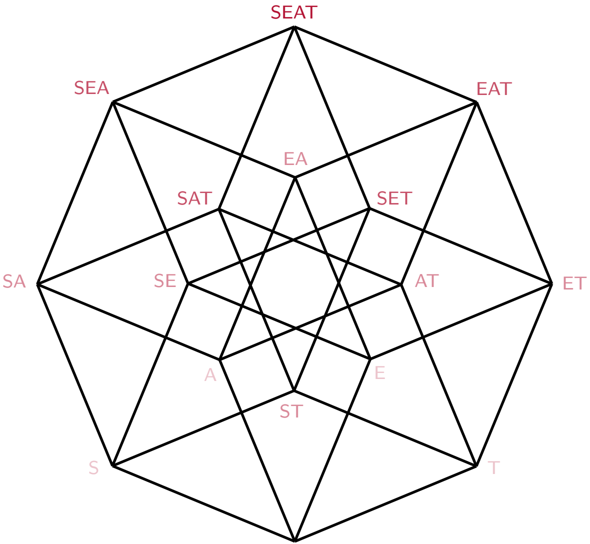
- Society
- Environment
- Activism
- Transformation



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marriage:	straight women	$\longleftrightarrow$	gay people
women showing flesh:	white women	$\longleftrightarrow$	Black women
gay men showing flesh:	white gay men	$\longleftrightarrow$	Black gay men
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3. Why we too often don't weigh up both cost and benefit.

	the math we want children to learn	⟷	the math-aversion it causes
	the tax we pay	⟷	the services society receives
attending a conference in person:	the harm we do	⟷	the good we do