Logic vs illogic: why mathematics is easy and life is hard

Eugenia Cheng

University of Cambridge

1st March, 2004

Approximate text of a talk given to the

Trinity Mathematical Society, Cambridge, on 1st March 2004

and to the

University of Chicago Mathematics Club on 14th February 2005.

Abstract

Generations of even well-educated adults are under the delusion that mathematics is difficult, and that mathematicians must therefore be very clever. In this talk I will explode this myth and show that mathematics is in fact easy, and moreover that mathematics is precisely "that which is easy" for an appropriate sense of 'easy'. We will take 'easy' to mean 'attainable by logical thought processes'. As a corollary, or a converse, or a contrapositive or something, we will discuss the importance of illogical thought processes in non-mathematical life and the fact that life is thus difficult. Hence or otherwise we will deduce that mathematics is not life, nor can it nor should it be.

Introduction

It is a fact universally acknowledged that mathematics is difficult.

If you tell someone you're a mathematician you invariably get one of two answers. Either "Oh, I never could understand maths" or "Wow, you must be really clever."

Of course, since mathematics *is* difficult it does follow that we mathematicians *must* all be *very clever*. As a mathematician, I am quite tempted to believe this. But *is it true*?

What I'm going to do today is take the bold step – perhaps rash step – of exploding the Myth of Mathematics. A bit like the Masked Magician whose TV show explained how magic tricks work – with the result that he was vilified by the Magical community. I am going to show that mathematics is *easy*, and in fact that it is *precisely* "that which is easy".

First I'm going to explain what I mean by "easy", which is "attainable by logical thought processes, without having to resort to imagination, guesswork, luck, gut feeling, convoluted interpretation, leaps of faith, blackmail, drugs or violence".

By contrast, life is hard, that is: it involves things that are *not* attainable by logical thought processes. This can be seen as either a temporarily necessary evil or an eternally beautiful truth. That is, *either* we can think that life is like that only because we haven't yet made ourselves logically powerful enough to understand it all, and that we should be continually striving for this ultimate rational goal. *Or*, we can recognise that we will never be able to encompass everything by rationality alone, and that this is a necessary and beautiful aspect of human existence.

In short: mathematics is necessarily easy, life is necessarily hard, and therefore mathematics is not life.

Here's the general plan for the talk:

1. Mathematics is easy
2. Ultimate rationality
3. Life is difficult
4. Mathematics is not life

1. Mathematics is easy

What is mathematics? This is a question that we don't ask enough, and we certainly don't answer it enough. I want to characterise it in terms of *purpose*: what is mathematics *for*?

Mathematics is there to make difficult things easier. There are lots of reasons that things can be difficult and mathematics doesn't deal with all of them (not directly, anyway).

Here are three *relevant* ways in which things can be difficult:

1. It might be that our intuition is *not strong enough* to work something out.
2. There might be *too much ambiguity* around making it impossible to work out what's what.
3. There might be *too many problems* to sort out and too little time in which to do it.

Mathematics then

- helps us to *construct* and *understand* arguments that are too difficult for ordinary intuition
- is a way of *eliminating ambiguity* so that we can know precisely what we're talking about
- cuts corners, answering a lot of questions at the same time by showing that they're all actually the same question.

How does it do it?

- by throwing away the things that cause ambiguity
- by ignoring any details that are irrelevant to the question in hand

You keep doing this throwing-out-and-ignoring, until you get to a point where all you have to do is apply unambiguous logical thought and nothing else.

Examples

1. a banana and a banana and a banana is three bananas
   a frog and a frog and a frog is three frogs
   etc
   hmm, something's going on here: " $1+1+1=3$ ".

2. Even better: 3 blondes and 2 brunettes is how many people?
   becomes: 3 people and 2 people is how many people?
   becomes: $3+2=?$

3. My father is twice as old as me but ten years ago he was three times as old as me.
   How old is he?
   or
   This bag has twice as many apples as that one but if I take ten out of each then this one has three times as many as that one.
   How many apples are in the bags?
   becomes
   $x=2y$ and $x-10=3(y-10)$
   – a simultaneous equation!

Now, in this case we might well have been able to do it without explicitly using simultaneous equations, but what about this next problem – can you do this one in your head?

A rope over the top of a fence has the same length on each side, and weighs one third of a pound per foot. On one end hangs a monkey holding a banana, and on the other end a weight equal to the weight of the monkey. The banana weighs 2 ounces per inch. The length of the rope in feet is the same as the age of the monkey, and the weight of the monkey in ounces is as much as the age of the monkey's mother. The combined ages of the monkey and its mother is 30 years. Half the weight of the monkey plus the weight of the banana is a quarter the sum of the weights of the rope and the weight. The monkey's mother is half as old as the monkey *will* be when it's three times as old as its mother was when she was half as old as the monkey will be when it's as old as its mother will be when she's four times as old as the monkey was when it was twice as old as its mother was when she was a third as old as the monkey was when it was as *old* as its mother was when she was three times as old as the monkey was when it was a quarter as old as it is now. How long is the banana?

4. I am very happy. How will I feel if I go bungee-jumping?

This has too many possibilities for ambiguity. So what does mathematics do with it? It ignores it! (Which makes it much easier.)

5. Does God play dice?

This again has too many possibilities for ambiguity.

6. Let S be the set of all sets that are not a member of themselves. Is S a member of itself?

This is what *Russell's paradox* is about, and again, we deal with this by ignoring it. That is, we declare that *in mathematics, we are not going to allow ourselves to think about such an S.*

7. We want to understand how playing snooker works. So first we imagine that everything is perfectly spherical, perfectly smooth and perfectly rigid. We might think about friction, coefficients of restitution, spin etc later. We can ignore irrelevant details like colour. Of course colour is not irrelevant in practice; but the added pressure of trying to pot the black to win is not a question that mathematics can deal with.

This is the crucial point – we make things easy by ignoring the things that are hard. We throw away the hard stuff. Mathematics is precisely all the stuff we don't have to throw away. The easy bits.

If maths is easy, why is it hard?

You might be wanting to point out a flaw in my argument already: if maths is easy, why does anyone find it hard?

Well, this isn't the fault of the mathematics. If someone finds maths hard it's probably because *nobody told them it was supposed to be easy*. And I'm not just being facetious. (That is, I'm not *just* being facetious.) If you look at something the wrong way up, it's *bound* to be harder. If you look at something the wrong way round as well, then it'll be *even* harder. If you then think you have to learn how to stand on your head and look in a mirror at the same time, it's no wonder that you give up and go and study English literature instead.

There are as many ways to make things difficult as there are to make them easy, and we can be sure that a whole ton of them have been applied to mathematics.

If someone finds maths hard it might also be because nobody told them what it was *for* – a fork is rather hard to use as a knife. It's also rather hard to use if you're trying to eat a sandwich. Or a bowl of soup. Or a packet of maltesers.

If someone finds maths hard it might also be because they have no desire to answer the question that the maths is simplifying. Trigonometry makes triangles really easy. But if you don't care about triangles you're unlikely to feel that your life has been made easier by trig.

But also, some people just *will* find things much harder if they're not *allowed* to use imagination, guesswork, or violence. Rationality says that this behaviour is to be deplored as we head towards the ideal of *ultimate rationality.*

2. The aim of ultimate rationality

Many people, especially mathematicians, philosophers and scientists, think that we as human beings should aim to become completely rational. That if we discover a way in which we're not rational, we should get rid of it, iron it out, in order to get closer to the goal of ultimate rationality. This has two facets:

1. We should *be* completely rational (that is behave rationally, think rationally).
2. We should be able to *understand* everything completely rationally.

I want to look at a little logic in order to work out what this might mean.

Background on logic

There's a Part II Logic question that goes like this:

> The beliefs of each member i of a finite non-empty set I of individuals are represented by a consistent, deductively closed set S_i of propositional formulae. Show that the set
>
> $$\{ t \mid \text{all members of } I \text{ believe } t \}$$
>
> is consistent and deductively closed. Is the set
>
> $$\{ \text{t} \mid \text{over half the members of } I \text{ believe } t \}$$
>
> consistent or deductively closed?

The point of the question is to show that *democracy doesn't work*. So what's this assumption we're using here about people's sets of beliefs?

> A set *S* of beliefs is (deductively) **closed** if anything you can logically deduce from the beliefs in *S* is also in *S*.

For example the set

{ all mathematicians are clever, I am a mathematician }

is not closed unless I believe "I am clever". The idea is that a rational, logical person will believe *anything that his beliefs logically imply.*

> A set *S* of beliefs is **consistent** if you can't deduce a contradiction from the beliefs in *S*. That is, you can't deduce both *p* and not *p* for any statement *p*.

For example the set

{ I am clever, I am not clever }

is clearly inconsistent, but so is

{ all mathematicians are clever, I am a mathematician, I am not clever }

and so is

{ all mathematicians are clever,
I am a mathematician,
nobody under the age of 50 is clever,
I am under the age of 50 }

A rational, logical person will not believe anything that *implies a contradiction*.

What is a rational person?

So a rational person's set of beliefs should be closed and consistent. It should also not include random beliefs in crazy things that have no basis. So everything should be rationally deduced. Nothing should rely on blind faith.

I'm now going to argue that *this idea of a rational person is absurd*.

3. Life is difficult

Life, frankly, is difficult.

The upshot is that rational thinking simply isn't good enough to cope with it. In this part of the talk I'm going to talk about the ways in which rationality *fails us* in life, by being:

- too slow
- too methodical
- too inflexible
- too weak
- too powerful
- and it has no starting point.

And that's *why* irrationality (or 'arationality') and illogic are not human weaknesses but human strengths.

Logic is too slow

In life, we don't always have time to go through logical thought processes to come to a decision. Emergency situations are much more urgent than that, and in that case the important thing is to make a decision that is *fast* rather than *accurate at all costs*. There's no point being right if you've already been flattened by the oncoming truck.

How do we know how to throw and catch? We don't have *time* to make clever calculations about projectile trajectories, we just do it. Some people are better at it than others but it has nothing to do with being *more logical* about it.

How do we sing in tune (if we do sing in tune)? We don't calculate the tension of the vocal chord that's required, to produce the correct frequency; in fact, we probably don't even know what the required frequency *is*. Well, some people are manifestly better at it than others but it has nothing to do with being more logical about it.

The speed issue is why we have reflex actions. We have *built in* reflex actions but we can also *train* reflex actions, like learning to say "You're welcome" automatically every time someone says "Thank you", or learning how to walk all the way to lectures even when you're still pretty much asleep.

Logic is too methodical

Logical thought proceeds calmly step by step through logical inferences. This isn't just slow, it's boring. You don't get into uncharted territory by taking little tiny safe baby steps. Remember that game *Grandmother's footsteps*?

The big leaps in life are the flashes of inspiration. *Flashes of inspiration* are nothing to do with logic. These happen both in mathematics and in other creative parts of life. The great geniuses of history are the ones who've made great leaps of inspiration. Now, inspiration in mathematics doesn't mean there's something about mathematics that isn't logical – it's just that in order to *make* bigger discoveries, a flash of inspiration gives us a big head start. It doesn't mean we need a flash of inspiration to *understand the result afterwards*, we just needed it to get there *in the first place*. It's harder to build a bridge across a river, than to cross the river once someone has already built the bridge. And while you're trying to build the bridge, it's helpful to be able to fly.

Logic is too inflexible

This is related to the previous point. Logic is too inflexible in the face of a flexible, and often rather random world. Logic is rigid and can't deal with that randomness.

Take evolution – a slow and logical process. But the great survival stories come from random mutations, some of which aren't successful but some of which are. Random mutation enables change and adaption much more quickly than evolution, like the good old biology GCSE case of those moths, who went a different colour in the city to be camouflaged against polluted tree trunks. Or something. I forget the exact details (as with most GCSE stuff). New deadly viruses appear, by spontaneously mutating from old ones. Bacteria develop resistance to antibiotics by spontaneous mutation. This is nothing to do with logic.

Take our use of language. We associate words to things, and essentially we're associating random sounds to notions. There's no logic to it at root. There *may* be some sense in the *etymology* of a word, but somewhere back in the history of the word is a random association that started the whole thing off. And we can do that because our brains have the *capacity* for random association. This is nothing to do with logic.

Logic is too weak

Another situation where logic can't help us is if there isn't enough information. To paraphrase – er – myself: the great thing about logic is that it eliminates the use of imagination and guesswork. And the terrible thing about logic is that it eliminates the use of imagination and guesswork.

There are an awful lot of situations in life where we don't have enough information to make a completely logical decision. Perhaps there is an unpredictable element, something random, something we can't detect, or things we just don't know, or we haven't got time and resources to find out. It's a bit like having to solve simultaneous equations where there are more variables than equations. You can pin down the solution set a bit but you can't get a unique solution out. And in life, there is usually an arbitrarily large number of variables and an extremely small number of well-defined equations.

What are we to do, just not make those decisions?

Instead, we do various things. We can

- go probabilistic
- go instinctive
- guess
- be random.

We can go probabilistic. For example, that doctor tells us that 99% of these operations are successful, so we go ahead with it. Or: most men wielding a machete in an urban public place are dangerous, so I'll get out of the way.

We can go instinctive: I don't like the look of this dark alleyway, I'll go a different way. Or: there's something too smooth about this used-car salesman, I don't think I'll trust him.

We can guess: like choosing lottery numbers. There's no logic there, but it makes some poeple exceedingly rich.

We can go random ourselves, and let the dice decide.

Decision making is indisputably *hard*. You try and gather more and more information but at some point your information (or your time) is going to run out, and *logic* is certainly not going to take you the rest of the way. It's just too weak. Now I'm not saying that you then have to make and *irrational* decision that goes *against* rationality, but you are going to have to make a *non-rational* or *arational* decision. Perhaps if something is pure logic, it doesn't count as a decision at all.

Logic is too powerful

Apart from the fact that logic is too weak, logic is also too powerful. Its unforgivingly brutal power forces us into extreme positions if we take it too seriously.

For example:

> It's ok to drink half a pint of beer in an evening.
> If it's ok to drink x pints of beer then it's ok to drink x pints and 1ml.
>
> In which case, it's ok to drink any number of pints in an evening.

In order to remain rational, that is, closed and consistent, we have to believe *either* that it's ok to drink any number of pints in an evening, *or* that it's not ok to drink *any* beer at all, ever.

Similarly but rather more controversially:

> It's ok to abort a pregnancy after 1 minute.
> If it's ok after x minutes then it's ok after x minutes and 0.1 seconds.
>
> In which case, it's ok to abort even after 8 and a half months.

What has happened? In order to remain "rational", we *either* have to be completely pro-life and say abortion is wrong no matter how early, or believe that abortion is always ok right up until birth. And yet plenty of "rational" people (including me) believe *neither of these facts.* What is going on?

The problem here is the subtlety of a *fine line*, or a *sliding scale*, or *grey area* between the black and the white. Somehow, we are able to deal with sliding scales in our heads, in a way that logic can't. It's like the straw that broke the camel's back. It brings me to Fuji's paradox:

Fuji's paradox

I've named this paradox after a Japanese bond trader called Fuji who first drew my attention to it. It's a case in point that he didn't even notice that there was a paradox at all.

It was back in the dark ages, before I realised that mathematics was easy, where bond trading is hard. So there I was, trading futures at Goldman Sachs, when this guy Fuji came along to tell us about the Japanese market. Now, Japanese interest rates are the lowest in the world, but Fuji's theory was that they would never actually hit 0 because then *everyone would know that they couldn't go any lower*. Because negative interest rates would be absurd.

> 1. Interest rates can't be negative
> 2. Interest rates will not be set at x% if everyone knows they can't go lower than x%

Now, the thing is that Japanese interest rates go in increments of quarter percentage points, so the Bank of Japan can only change rates by multiples of that. So, I thought to myself, if Fuji's theory is right then interest rates will not be set at 0.25% either, because then everyone will know it can't go any lower, since it also can't be 0. Oh, but then it can't be 0.5% either. Nor can it be 0.75%, or 1%...which means that it can't be *any* percent – which means that Japan can't have interest rates!

This is clearly not true – Japan did have interest rates and still does. So what's gone wrong? (Actually a couple of years later, Japanese interest rates really did go negative, but that's another unbelievable story)

This is in fact a manifestation of the "unexpected hanging" paradox.

The paradox of the unexpected hanging

You will be hanged sometime this week but on a day when you're not expecting it...

The prisoner is told that he will be hanged sometime this week but on a day when he isn't expecting it. So he thinks to himself: well it can't be on Sunday, because if I hadn't been hanged by Saturday then I'd know that it had to be Sunday, so I'd be expecting it. So it has to be Saturday at the latest. But then it can't be *on* Saturday, because if I hadn't been hanged by Friday and it can't be Sunday, then I'd know it was Saturday and I'd be expecting it. So it can't be Saturday...so it can't be Friday...or Thursday...or Wednesday...or Tuesday...or Monday – which means I won't be hanged!

And then on Monday he is hanged, and he really isn't expecting it.

We can only imagine how miffed he feels, hanging there trying to work out where his logic went wrong.

Logic has no starting point

Well, my last objection to logic is that it has no starting point.

If we're not going to take anything on blind faith, we're simply not going to get anywhere. You can't prove something from nothing; you can't deduce anything from nothing; there's no such thing as a free lunch.

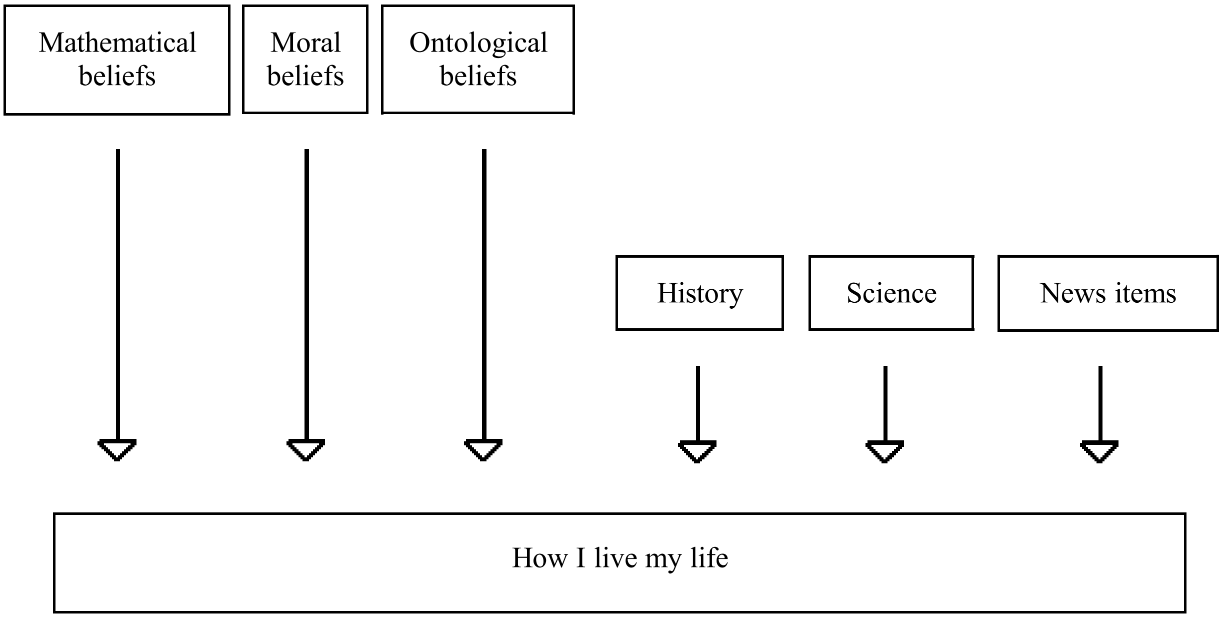
This seems to me to be an obvious and immediate flaw to the idea of the ultimately rational person. Does anybody here think that they don't believe anything at all?

But does that mean we should immediately and completely give up? And yet, there's still some scope for greater and lesser rationality here. For example

- a rational person is supposed to believe that the sun will rise every morning
- a rational person is supposed to believe that 1+1 = 2
- a rational person is not supposed to believe in ghosts
- a rational person is not supposed to believe in ESP
- is a rational person supposed to believe in God?

Where do these "supposed"s come from? They come from society. It hasn't always been the norm to believe that the sun will rise every morning. And in some societies it is the rational norm to believe in God and in others it is not. So in fact, rationality is a sociological notion.

When I was thinking about this I decided to sit down with the small task of writing down everything I believe, and drawing a picture to show what depends on what. I came up with the following types of belief: Mathematical, Moral, Ontological. I found I also believe in certain events in history, certain scientific facts, and certain news items. I also have various beliefs about how I live my life. I organised these according to the following schematic diagram, showing that in fact my beliefs about how to live my life probably follow from the other more "basic" beliefs. But also, some of my basic beliefs feel more basic to me than others, which is why they're further towards the top:



Why do I believe these things? These beliefs come from my education and from society in general.

So apparently you can still count as rational as long as all your basic premisses come from the big bank of basic premisses accepted by society as 'rational things to believe'. If the things up at the top are "the moon is made of soft green cheese" and "sleeping upside down is good for the elbows" then the men in white coats will soon come and take you away.

But still, I've had arguments with people (mainly philosophers) who get very upset if something I'm defending comes down to something I believe, and I declare that I believe it 'because I do'. Rational people aren't supposed to do that are they? Would it be better to be entirely circular? Are they supposed to be Big Bang types? Where nothing ever actually comes down to an explicitly specified belief?

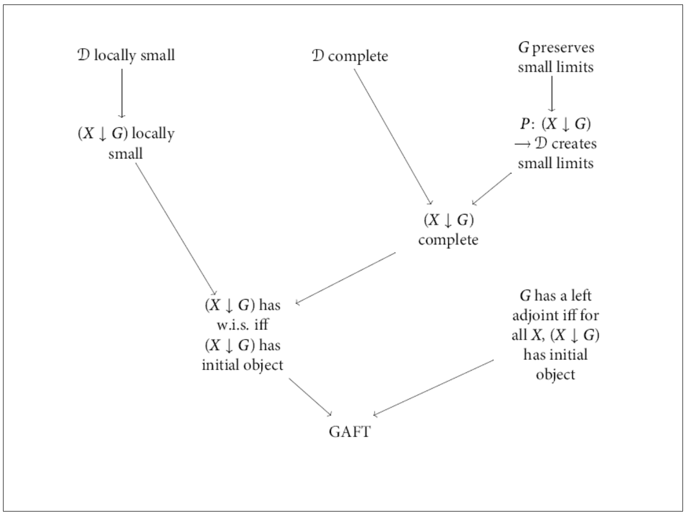
Well *I* believe it's a good thing to be aware of what you're assuming. I repeat:

I *believe...*

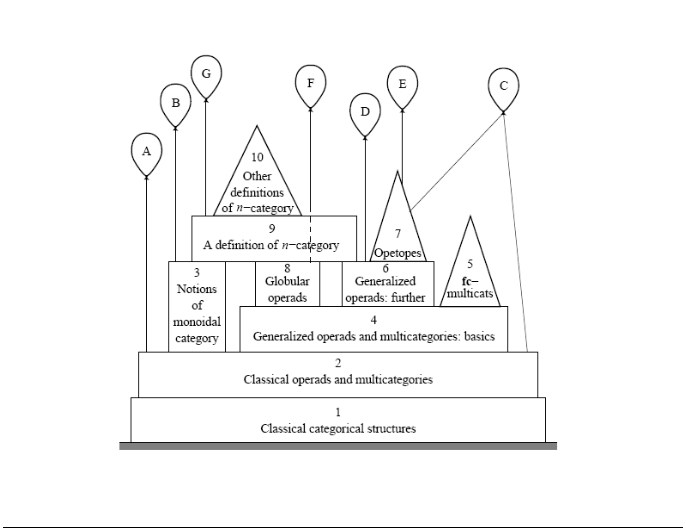
...it's a good thing to be aware of what you're assuming. Whether it's a whole lot of things up there at the top of your diagram, or, say, God.

Being aware of your assumptions definitely part of the discipline of mathematics, and part of what makes maths *easy* – everyone has to state very clearly what their basic premisses are. A good way of pointlessly losing marks in an exam is by not stating clearly the assumptions that are being made in a proof. Conversely, a good way of making a proof very clear is by highlighting *where* and *how* each of the assumptions is being used. A good way of understanding a proof or any kind of argument is to draw a picture of dependencies.

Here is such a picture for the General Adjoint Functor Theorem (from Category Theory):



Here is the structure diagram from Tom Leinster's book "Higher operads, higher categories":



What if we drew a picture for the whole of mathematics? What is there up at the top here, or is there one God of mathematics? Perhaps there is only one thing that we have to believe: Modus Ponens? How big a leap of faith *is* that?

Here is a paradox from Lewis Carroll: does statement Z below follow from A and B?

The Carroll Paradox
from: *What the Tortoise Said to Achilles*
transcribed as *Two-Part Invention* by Douglas Hofstadter

(A) Things that are equal to the same are equal to each other.
(B) The two sides of this Triangle are things that are equal to the same.
(C) If A and B are true, Z must be true.
(D) If A and B and C are true, Z must be true.
(E) If A and B and C and D are true, Z must be true.

⋮

(Z) The two sides of this Triangle are equal to each other.

Does Z follow from A and B? It does seem to, but only because we seem to be accepting C. Does Z then follow from A and B and C? It does seem to, but only because we seem to be accepting D. Does Z then follow from A and B and C and D? Oh dear.

In order to get from A and B to Z directly, we basically need to invoke *Modus Ponens,* the rule of inference which says if we know p and we also know p *implies* q, then we know q. I claim that this is the basic belief from which all of mathematics begins. We have to believe it in order to get anywhere.

4. Mathematics is not life

Can we and should we aim to be completely rational?

Can we possibly hope to encompass *everything* in the realm of rationality? And *should* we?

Let's consider each of these ways in which life is too difficult for logic and ask three questions:

1. Can we iron this sort of problem out of existence? That is, decide that the problem *itself* is illogical and so we shouldn't have to deal with it
2. Do we want to iron this sort of problem out of existence? Even if we *can* declare the *problem* to be the problem, do we want to just ignore it?
3. Can we change our notion of logic so that logic *can* deal with it?

Logic is too slow

Remember this was about emergency situations. We can't just get rid of emergency situations (although we do try to prevent them if we can). Perhaps actually, *training one's reflexes* is a question of training oneself so that the *processes* of logic happen extremely quickly.

Let's take a survey. Consider the following argument.

There's a fire. I must get out!

Is that a logical process?

It certainly wouldn't count as one if we tried to write it down in formal logic, in which case it would look like this.

> p
>
> q

We would have to add in at least one axiom

a = "if there is a fire I must get out" ie $p \Rightarrow q$

I would prefer to add a few more to make it logical:

> There is a fire.
>
> If there is a fire and I do not get out, then I will burn.
>
> I do not want to burn.
>
> I must get out.

We could make this look a bit more like mathematics like this:

> "there is a fire" and "I do not get out" imply "I get burnt"
> ie p and not $q \Rightarrow x$
>
> "I get burnt" is bad and to be avoided
> ie $x \Rightarrow$ bad
>
> $p \Rightarrow q$
>
> q

If you've done formal logic you'll see that there we're still a long way away from a formal proof. But if we're just thinking of rational argument, I would call that a rational argument, where the previous "*p, q* " attempt was not exactly "rational" though also not irrational.

But certainly

There is a fire. I will stay here.

is a rather irrational process. So I suppose we need to be careful here about the difference between *irrational* and *arational*.

To deal with this case we could make emergencies into logical decisions by adding various beliefs into our belief system (assuming we're allowed some beliefs). So if we put $p \Rightarrow q$ in this case, it does turn into something rational.

This is just like using a lemma to make a proof shorter. So training our reflexes might be a bit like adding useful lemmas that we don't *have* to prove every single time we use them.

Logic is too methodical/inflexible

Here we're talking about baby steps vs leaps of inspiration and random processes. It seems to me that leaps of inspiration are an exceedingly important part of human progress, as are random processes. And anyway, randomness is *fun*. Apart from the chance of winning the lottery, wouldn't life be boring if it were completely unrandom?

Logic is too weak

– in the face of incomplete information. Well, we're always going to find ourselves in situations where the information available is incomplete. It's true that you might not want to call it *irrational* if you're necessarily taking all the information available and doing the *most* *rational possible thing*. But if you have to make a guess or a leap of faith at some point, it simply isn't completely rational. If you did that in a mathematical proof you would get a lot of complaints. Perhaps we need to extend our notion of logic to include this as a possibility?

What's the idea – that something is a *logically plausible* deduction if it doesn't result in a contradiction. So, given all the information, there might be a whole range of *logically plausible* results – and then you're allowed to pick any of them.

Similarly: when you have a system of linear equations with more variables than equations, you might be allowed to pick any solution that's in the solution space.

Or, more 'lifelikely': suppose you want to pick something off a restaurant menu.

Marinated Roast Breast of Chicken - £12.50
with caramelised pineapple, coconut sauce
and wild rice

Pan Fried Ostrich - £14.00
set on a bed of a ratatouille
with a rosemary and redcurrant jus

Char Grilled Fillet Steak - £16.95
with stilton and red wine sauce

Smoked Finnan Haddock Fish Cake - £12.95
with spinach, poached egg, sorrel sauce and fries

Roasted Summer Vegetable Tart - £11.95
with basil oil dressing

All mains are served with a medley of fresh seasonal vegetables

Of course, you don't actually know what anything is going to taste like. But there are some parameters:

– you don't want to spend more than fifteen pounds
– you don't like fish

which still leaves the chicken, the tart and the, er, ostrich – at which point you're going to have to do something random/inspired/intuitive. But not anti-rational; choosing the haddock on the other hand would be anti-rational.

However, this again is not something we're allowed to do in mathematics – much as we'd like to. A lack of a counterexample is not a proof, although checking that a result does not yield inconsistencies is a good start.

Logic is too powerful

This was the problem of fine lines, grey areas, sliding scales

Now I personally run into a lot of trouble over *fine lines*. I'm a great believer in fine lines and grey areas. I *don*'t feel the need to be pushed to the extremes, and I'm *not* afraid of the inconsistencies that arise. You see, there are two ways to remedy the fine line problem: stop trying to be deductively closed and stop trying to be consistent.

If you've ever had an argument with me you might have noticed that I've given up both of these terrible vices. It doesn't mean I go round believing statements and their negations at the same time – if you give up deductive closure then giving up consistency isn't quite so bad. I might believe things that *imply* both p and not p, but since I'm not closed, it doesn't mean I actually *believe* p and not p. This deals with both the beer example and the abortion question. Perhaps I should emphasise that I take *none* of the extreme positions suggested earlier by the "ultimate rationality" arguments.

> It's ok to drink half a pint of beer in an evening.
> If it's ok to drink x pints of beer then it's ok to drink x pints and 1ml.
>
> In which case, *rationally*, it's ok to drink any number of pints in an evening.

However: *just because it's logically implied, doesn't mean I'm going to believe it.*

I do concede that it must be rather frustrating to try to have an argument with someone who is completely unphased by being logically inconsistent. But *seriously*, is strict logic a good enough reason for believing something, or would it be more interesting to examine why the strict logic has gone wrong?

There is another possibility if you feel queasy about going non-closed or non-consistent: something I'd like to call quantum logic but I can't, because that's already been bagged for something else. Instead, I'll call it Local Logic, or Temporal Logic because it adds a time factor. It says that *if I say I believe something, it only means I believe it now.*

Is it contradictory to believe one thing yesterday and the opposite today? Not really, because you're not really believing both those things at the same time.

We have a set of beliefs, but we can't ever work out what the *whole* set of beliefs is, because the process of probing the set of beliefs *changes the beliefs*. (Which is why I wanted to use the word 'quantum'.) We've probably all experienced an argument with a philosopher in which we end up saying we believe something that we didn't believe an hour ago – and that we won't believe an hour later.

I believe in free will, until someone comes along and asks me very difficult questions about it, in which case I will end up declaring that I don't believe it. Philosophers can argue with me convincingly to persuade me that I can't *possibly* believe that numbers exist – but once they've gone away I'll settle back to my usual belief that numbers *really do exist*.

So the only way of getting at two beliefs simultaneously is to have a third belief asserting the simultaneous belief of the first two.

We could imagine using ordinary logic and adding times – instead of

$$\frac{p \quad q}{p \text{ and } q}$$

we'd have to have

$$\frac{\{\ p \text{ at 9pm }\} \text{ and } \{\ q \text{ at 9pm }\}}{p \text{ and } q \text{ at 9pm}}$$

except that by the time we've written that down it's 9.01pm and we have to do it again.

Logic has no starting point

Finally, what of the problem that logic has no starting point?

We simply have to believe in some things. But a very important and valid activity is the process of working out exactly what those initial premisses are. And we can aim to become *more rational* in the sense of having as few inital premisses as possible, and certainly, being aware of what these are. This brings me to the last part.

Mathematics is not life

So: maths is easy, life is hard, therefore maths isn't life.

This doesn't mean that we shouldn't try to *extend* the scope of mathematics for it to include as much as possible, just as we should try to become 'more and more rational' by continuing to work out what the inital premisses of our beliefs are. The pursuit of mathematics is "the process of working out exactly what *is* easy, and the process of making as many things easy as possible".

Definition

Mathematics is
"the process of working out exactly what is easy, and the process of making as many things easy as possible"

But we should *not* feel affronted by the existence of things that *can't* be subsumed by mathematics, the irrational or arational, the illogic. Without that there would be no language, no communication, no poetry, no art, no fun.

The essence of many beautiful aspects of human life comes in that interface between the rational and the irrational, the counterpoint between logic and illogic. Logical processes fail to deal completely with life – in the face of speed, chance, lack of information. And this interface intrigues us in the games that even the most logical and rational of people like to play

- bridge/backgammon which pits logic against a carefully controlled element of chance
- chess/go/golf which pits logic against a backdrop of information which can with skill be tapped, but never completely
- team/opponent sports which pit logic (aka tactics) against speed

And finally, there's music, in which the essence of beauty lies, I think, in the delicate interplay between logic and flashes of inspiration.

I'd like to end with this little advert for arationality. If you publically endorse holding inconsistent beliefs, if you see no reason to believe the logical implications of all your beliefs, and if you confidently hold beliefs as inital premisses with no logical justification – in short, if you embrace arationality as a glorious part of human life, you will be logically invincible, as there is no way of logically defeating an arational argument. Try it. You will probably find yourself attacked for *not* being rational, which your opponents will find *particularly* unforgivable if you're a mathematician.

But then, we mathematicians know that ultimate rationality is only possible within mathematics, and there's more to life than that.

> *Logic is as empirical as geometry.*
>
> H. Putnam