The maths of popping open a bottle of bubbly

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Abstract

In this festive note we study the sound of a cork pop from a bottle of bubbly, as a tool for elucidating some aspects of fourier analysis, the physics of sound, logarithms, and exponential decay.

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1 Introduction

Our aim is a festive study of the enticing "pop" of a bottle of bubbly being opened. There are many ways to open sparkling wine, from a loud bang spraying bubbles everywhere, to the near silent "pff" as executed by smooth experts. What can maths tell us about any of this?

We analysed the different types of pop produced with two variables

1. temperature, and
2. hand pressure.

To quantify the amount of "ring" to the pop, we recorded the sounds and analysed them using computer software. We used this analysis to identify a quantity which might correspond to the amount of ring in a pop, and found a relationship between temperature, hand pressure, and ring.

2 Background

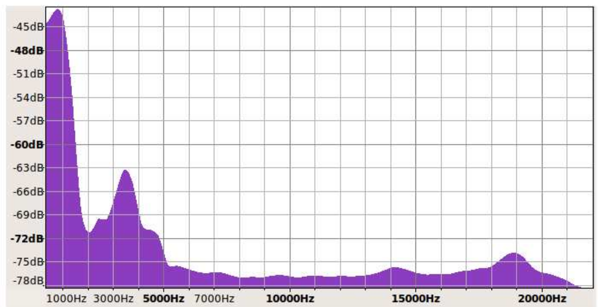
2.1 Timbre

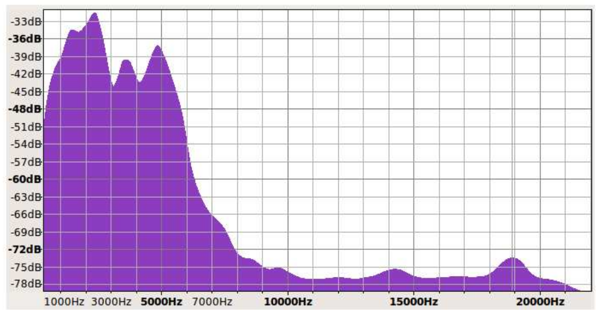
Timbre refers to the tone quality of a sound. Other than computer generated digital sounds, all sounds are comprised of a range of different pitches superimposed on top of each other. When something sounds like an actual note with pitch, the pitch we hear is just the base or "fundamental" pitch, and the higher pitches are perceived by our brain as blending in with the fundamental pitch to produce different timbres. The proportions of higher pitches blending in are what create different timbres to our ears. For example, the difference between a violin and a flute playing the same note can be seen by analysing the proportions of the different higher pitches blending in with the fundamental note.

Fourier analysis is a technique in mathematics whereby any wave can be decomposed into constituent sine waves. A pure sine wave will only be produced by a digital beep; other less "perfect" shaped waves will be made by adding together different sine waves. Sine waves of different frequencies combine to make a more wonky looking wave with an overall frequency derived from the individual frequencies. This corresponds to the way in which different pitches combine to make the timbre of a note. The frequency of the constituent sine waves corresponds to their pitches, and their relative amplitudes show how much each higher pitch is contributing to the overall shape of the wave.

The shape of the sound wave can only really be seen if a continuous note is being played, unlike the pop of a champagne bottle. However computer software performs "spectral analysis" on a sound sample and measures the loudness of the various pitches contributing to the sound. The graph produced in this way is a graph of loudness against pitch. Loundess is measured in decibels and pitch is measured by frequency, in hertz. Here are some examples for comparison.

The first graph is me singing an oooh as beautifully as I can and the next is me screeching. In the first one you can see a clear extra peak in the graph, whereas in the second one there is a dense cluster instead.





The range of human hearing is around 20 Hz to 20,000 Hz. Texts on sound engineering variously place the ringing sound of bells and cymbals is in the 8000 Hz to 12,000 Hz range. For example the Mixing Engineer's Handbook (Owsinski) places "brilliance" in the 6000 to 16000 range and "sparkle" (of cymbals) in the 8000 to 10000 range.

Note that the scale on the vertical axis is negative for a technical reason; the software measures decibels below the reference point 0 which is taken to be the maximum level the system can handle. Thus these are not absolute measures of loudness. Also note that decibels are measured on a logarithmic scale. This means that if something becomes 10 times louder we don't multiply the decibels by 10, but rather, we add 10.

2.2 Sound and pressure

Sound waves are created by changes in pressure in the air molecules. The sound of a cork popping is made by the sudden escape of pressurised gas from the inside of the bottle. That escape is affected by the pressure from the gas inside pushing out, and the pressure from our hand from the outside pushing down on the cork.

2.3 Temperature

The pressure of the gas in the bottle is affected by the temperature of the liquid, as gas has less energy at lower temperatures so exerts less pressure.

The temperature is in turn determined by the cooling time (and method) and the relationship is exponential. Thus if you increase the cooling time in 10 minute increments the temperature decreases more quickly at the beginning and more slowly at the end. This is because the temperature difference between the chosen cooling environment and the liquid decreases as the liquid cools, so it cools more slowly as it gets cooler.

We achieved the following results for chilling in an ice bucket:

time (minutes)	temperature (C)
0	21
10	15.2
20	11.9
30	9.4
40	6.7

We can approximate this with the following equation. Let T be temperature and t be time, then

$$T \approx 21e^{-0.03t}.$$

However in the end we decided to stick with the time in minutes, although this has the unsatisfactory effect of having to impose boundary conditions on the formula as it doesn't stabilise with time; we realised we would need boundary conditions on the hand pressure in any case (see below).

2.4 Chilling methods

The simplest way to chill champagne is in the fridge, but it is hard to control the temperature this way. Apparently the ideal temperature is between 6 and 7 degrees celsius; my fridge only gets the wine down to about 11C. So I used the ice bucket method, where the bottle is submerged in a mixture of half ice and half water. The bottle was chilled to the optimal temperature after 40 mins so I investigated 10 minute increments in between. To take the temperature I used a digital thermometer immediately after opening the bottle. The exponential cooling was clear, but in the end I decided to use chilling time as a variable rather than temperature, because for most normal people there's no good way to measure temperature except by opening the bottle. (Professionals have precise temperature controls.)

2.5 Hand pressure

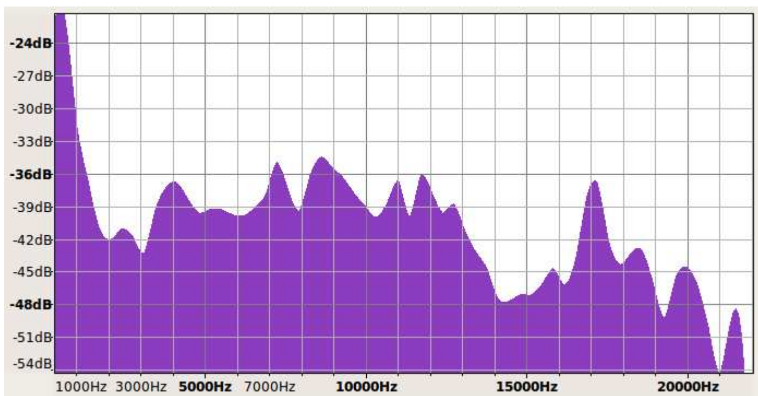
Pressure is force divided by area, but the area of the bottle is constant so the pressure is proportional to the force. I measured my downward force by putting the bottle on a digital scale and taking a reading as I pressed down. The scale produces a reading in Kg which is technically mass, but as we were operating in more or less constant gravity this also comes down to measuring force.

We need boundary conditions on hand pressure because of course if too much pressure is applied the cork won't come out at all.

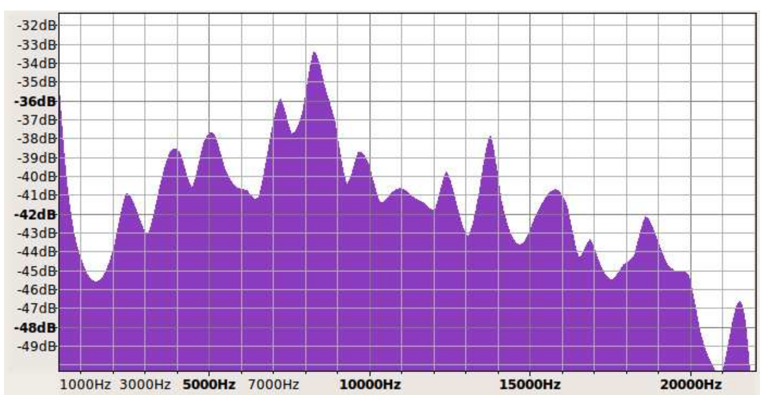
3 Results

The most ringing pops were, unsurprisingly, from the bottles that were well chilled and opened without any resistance. This was never really in question: the question was how this timbre difference would appear on the graphs.

Here are some sample graphs, from a ringing pop:



and from a dull "pff" (note different scale on the vertical axis)



The clear general pattern was that the dull "pff" type pops had a big peak around 8000 Hz and then dropped off, whereas the ringing pops continued to have peaks to more or less the same level up to around 12000 Hz or so. So I decided to take the difference between the decibel level of the 8000 Hz peak and the level around 12000 Hz as a measure of the "ring" of the pop. This matches what we mentioned earlier about "brilliance" and "sparkle.

As a smaller difference means a nice ringing pop, we decided to subtract this difference from 10 to get a rating for the ring of the pop. Thus if the difference in decibel levels is 0, the pop scores 10 out of 10 on ring.

Definition.

Let D_{8000} be the decibel level on the spectral analysis at the 8000 Hz peak, and D_{12000} the decibel level at the 12000 Hz peak. Then we define the ring of the pop to be:

$$\text{ring} = 10 - (D_{8000} - D_{12000}).$$

Note that the software determined dB values to 1 decimal place.

Then we use the following variables:

- Let P be the pressure of the hand on the cork, measured in Kg.
- Let T be the time in minutes that the bottle was chilling in an ice bucket, starting from room temperature.

There was a clear pattern that the numbers increase as pressure increases and as chilling time decreases. We model this using the following approximate relationship within the range $0 \leq T \leq 40$ and $0 \leq P \leq 12$.

$$\text{ring} = 7 + \frac{T}{40} - \frac{P}{3}.$$

We have the following table of experimental results compared with predicted results:

key:

test number	d 8000 (-dB)	ring (formula)
d8000-d12000	d 12000 (-dB)	ring (experiment)

pressure (kg)	12	p14	33.4	3.00			3.25			3.50			3.75			4.00
		7.2	40.6	2.8												
	9	p03	30.8	4.00			4.25	p08	34.8	4.50			4.75	p13	34.0	5.00
		6.2	37.0	3.8				**5.2**	40.0	4.8				**4.4**	38.4	5.6
	3	p01	35.9	6.00	p04	33.4	6.75	p05	33.3	6.50			6.75	p07	31.8	7.00
		4.4	40.3	5.6	**3.5**	36.9	6.5	**3.1**	36.4	6.9				**2.8**	34.6	7.2
	0	p02	33.1	7.00			7.25	p09	32.9	7.50	p06	36.2	7.25	p11	34.4	8.00
		3.9	37.0	6.1				**3.1**	36.0	6.9	**2.8**	39.0	7.2	**1.6**	36.0	8.4
		0			10			20			30			40		
		chilling time (mins)														

Note on dimensions

Humourless observers may note that the dimensions of this formula are a bit suspect. However, I think the idea is that T and P are each proxies for a measurement of pressure, with the two values applying in opposite directions (hence one negative sign). Decibels measure sound level by taking the logarithm of a ratio of pressures. Beyond this further investigation into the physics of champagne corks would be needed. Physics often starts by trying to model a situation with a formula that produces the desired results; it is a step beyond that to understand *why* that formula works.

And my excellent statistics teacher Mr Muddle used to say, the outcome of a test is always "Further research, with further funding, is required."

4 What is the perfect pop?

It should be noted that different pops are desirable for different reasons. A nice loud ringing pop might be fun for a celebration, but the pop is caused by a sudden escape of gas which means that the wine will lose a lot of fizz and possibly some flavour in the gas. Thus experts aim to open the bottle with as little noise as possible. This turns out to take a large amount of force on the cork while opening it.

There are other methods of opening a champagne bottle. For example, you can actively push up on the cork to make it fly a long way. You can also take a sword and slash the top of the bottle off. Neither of these methods is recommended because of the risk of injury. I have actually been injured by the top of a bottle of sparkling wine that was slashed off with a sword. The cut bottle top bounced on the floor and then sliced a cut in my leg. The cut was a beautiful parabola showing the trajectory of an object moving under the force of gravity; that however is the subject of a different study.

Finally it should hardly need pointing out (but it does) that this study was done in a spirit of fun and was not under scientifically controlled conditions. A total of 16 bottles was tested. Cheers.